Heterogeneous firms, vertical linkage and horizontal FDI.

Léo LE MENER†

December, 2008

Abstract

Introducing intermediate goods is a well known way to explain vertical Foreign Direct Investment (FDI), however this paper argues that the characteristics of intermediate goods are also a decisive element for horizontal FDI, even when countries are symmetric, and particularly when the downstream sector is closely related to the upstream one. This paper extends Melitz, Helpman and Yeaple (2004) model by introducing an upstream sector. Two production factors are then introduced in the production function of the final good: labour which depends on the productivity of firms, and the intermediate good which depends on the structure of the assumed fixed proportion technology. In order to serve foreign markets, heterogeneous firms of the downstream sector can either export or invest abroad. The paper shows that the characteristics of the upstream sector modify the results obtained by Melitz et al. First, they affect the distribution of the firm’s productivity and attenuate the impact of heterogeneity on output level differences between firms with consequences for the firm’s choice to export and invest. Finally, the paper looks more particularly at a specific sector, the agrifood industry which is closely related to the agricultural sector. The perishability of agricultural products increases trade costs. The more perishable the products the higher the trade costs, and thus the more dependent on local agricultural supply agrifood firms are. This unique connection between sectors affects the relationship between the productivity level and profitability of firms, and thus their ability to serve foreign markets and the way they proceed, by export or FDI.

Keywords: Horizontal Foreign Direct Investment, firm heterogeneity, vertical linkage, international trade theory, intermediate good.
1 Introduction

In the last decade a new approach to the analysis of cross-border trade and cross-border investment, triggered by empirical observation (Bernard and Jensen, 1995), has exploded. This new theoretical framework was developed by Melitz (2003), Helpman, Melitz and Yeaple (2004), Bernard, Eaton et al. (2003) among others, and has resulted in new ways of thinking about firm heterogeneity and participation in international markets. One dimension which has received particular attention is the relationship between firm level productivity and entry to and survival in export and FDI market\textsuperscript{1}. The key point of this literature is that the interaction of sunk costs and productivity heterogeneity is a determinant of why some firms stay in their own domestic market, why some firms export, or why some others invest abroad. In these theoretical models, there exists some endogenously determined threshold that determines which firms do and do not produce, export and invest abroad. The core Melitz (2003) model is now being developed in various ways. It provides a new approach and new results compared to the Krugman model on which is based.

Chaney (2008) uses the Melitz core model to divide the aggregate gravity trade flow into two margins, an extensive one, which is the growth of the number of exporters, and the intensive margin, which is the export volume per firm. He shows that the intensive and the extensive margins to trade are affected by the elasticity of substitution into two different way. The extensive margin to trade dominates and the impact of trade barriers on trade flows is reduced by the elasticity of substitution, so the results of the Krugman model are overturned.

Helpman, Melitz and Yeaple (2004) extend the Brainard (1993, 1997) models of trade-offs between proximity and concentration, introducing Melitz's firm heterogeneity, to consider the decision to set up an overseas affiliate. They build a theoretical model based on Melitz (2003), and they econometrically test their predictions. They find that export sales relative to foreign affiliates sales is negatively impacted by the heterogeneity of the domestic sector, so when the heterogeneity of firms is higher, or when the elasticity of substitution is higher, FDI sales are more important relative to export sales. The econometric results support the theoretical predictions.

In the basic firm heterogeneity model, there is only one factor of production, which is labor. But recently, models with two factors of production have been developed, as Bernard (2007). In this model with skilled and unskilled workers, the Melitz model is combined with Hecksher-Ohlin factor endowment differences. The aim of this paper is to see the effects of asymmetric countries on factor reallocation and aggregate productivity. They find that the heterogeneity of firms magnifies ex-ante comparative advantage and provides a new source of welfare gains from trade.

Toubal and Kleinert (2006) look at the impact of distance costs on the average sales of foreign affiliates. They build two models, one with symmetric firms and a specific input, and one with heterogeneous firms but without intermediate inputs. They found that aggregate affiliates' sales fall in distance, and that, in the heterogeneous firms model, distance negatively affects the number of affiliates. In their appendix, a unified model with specific intermediate inputs and heterogeneous firms delivers ambiguous results on the impact of distance costs on the average sales of foreign affiliates.

Bombarda (2008) built a model with heterogeneous firms and intermediate goods to investigate the role of distance when, due to proprietary technological issues, one of the intermediate goods must be produced at home. She shed light on the non-monotonic relation between distance and the FDI supply-mode cutoff.

\textsuperscript{1}For a critical review of this new literature, see Greenaway (2007).
Even models with an intermediate good don’t investigate the pure intermediate good effects on productivity or threshold in the final good industry, but rather they investigate how the presence of intermediate goods mediates other measures.

Our intuition is that firms are affected by the characteristics of their inputs and also by the nature of the good they produce. The specific relationship between a final sector and an upstream one could affect the firm’s ability to serve foreign markets and the way they proceed, by export or FDI. This specific relationship is consequential for some sectors such as the agrifood industry, which is closely related to agricultural sector. The fact that there is a linkage between the agricultural sector and agrifood sector and that this linkage is fundamental to the activities of agribusiness firms could have important implications for the future of European agribusiness activities.

Changes of European agricultural policy may affect the location of agricultural activities, agricultural prices, and the structure of the European agricultural sector. This raises many questions. For example, changes in the location of agricultural production at the European level (Chevassus-Lozza and Daniel, 2006) can affect final good production and investment in each European country. Moreover, changes in agricultural productivity and price due to CAP reforms (Lips and Rieder, 2005) can affect the attractiveness of the European Union for agribusiness FDI from other countries, and can affect the productivity of European agribusinesses. Then, the impact of CAP reforms on agribusiness may not be uniform as those agribusiness firms which are more closely related to the agricultural sector may be more affected.

The purpose of this paper is to build a basic model of vertically linked heterogeneous final good sector firms to provide a basic framework to analyze the possible effects of the liberalization of the agricultural sector on agribusiness firms. Moreover, the approach could be generalized to all activities which are closely related to an upstream sector, when the final product is largely composed of a single input.

To do so, we extend the Melitz model with heterogeneous firms by introducing an upstream sector. The linkage between the final good sector and the intermediate good sector is made via a fixed proportion technology, so whatever the final good production level, firms need a constant fraction of the intermediate good to produce one unit of the final good. To focus on the effects of the vertical linkage, the upstream sector is reduced to a simple form: a representative firm, producing in perfect competition. The efficiency of the upstream sector will determine the price of the intermediate good, so there are two variables added with respect to the Melitz model. They are the intensity of the final good in intermediate good and the price of this intermediate good. The aim of this paper is to investigate the effects of these two variables on the core Melitz model. To go further, we introduce the option for final sector firms to serve foreign markets by export or Horizontal Foreign Direct Investment (HFDI) following the Helpman, Melitz and Yeaple theoretical model.

In the first part, we set-up the model and give some results in a closed economy. In the second part, we analyze the effect of intermediate goods in a open economy, first only with exports, and then with a trade-off between exports and horizontal FDI. The last part concludes and gives some ideas on the way that this model could be extended.

2 Set-up of the model

This model is based on Melitz (2003) and Helpman et al. (2004). We consider a world with two vertically related sectors. The intermediate sector uses labor to produce a homogeneous good,
and the final sector produces a differentiated good with labor and the intermediate good. The intermediate good is used entirely by the final sector, so the representative consumer only consumes the final good.

The quantity of intermediate good used to produce one unit of final good is exogenously determined by the nature of the good and the final sector activity, so there exists a technological constraint on the composition of the final good. For a given sector, all firms use the same quantity of homogeneous good to produce one unit of differentiated good. As in Melitz and Helpman-Melitz-Yeaple, firms in the final sector are heterogeneous, so the quantity of labor used by a firm to produce one unit of final good is a decreasing function of its productivity.

The amount of labor available in the economy is inelastically given at its aggregate level by the size of the country, and it’s used by the two sectors.

2.1 Demand

The preferences of a representative consumer are given by a C.E.S. utility function over a continuum of goods indexed by $\omega$:

$$U = \left[ \int_{\omega \in \Omega} y(\omega)^\rho d\omega \right]^{1/\rho} \tag{1}$$

This utility function only depends on final good consumption and $\Omega$ represents the set of available varieties. Varieties are substitutes, which implies that $0 < \rho < 1$, and the elasticity of substitution between any two varieties is given by $\sigma = \frac{1}{1-\rho} > 1$. As in the Dixit-Stiglitz model, we can consider the set of varieties consumed as an aggregated good $Y \equiv U$ associated with an aggregated price $P$.

Optimization of consumer preferences leads to the optimal consumption of each variety $\omega$:

$$y(\omega) = \frac{p(\omega)^{-\sigma}}{\int p(\omega)^{-\sigma} d\omega} R$$

which could be written with the aggregated price index $P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}$

$$y(\omega) = Y \left( \frac{p(\omega)}{P} \right)^{-\sigma} \tag{2}$$

And we also have the expenditure for each variety which is given by

$$r(\omega) = R \left( \frac{p(\omega)}{P} \right)^{1-\sigma} \tag{3}$$

where $R$ is aggregate expenditure. These results are the same as Melitz, and they are standard in monopolistic competition.

2.2 Production

2.2.1 Intermediate good sector

The intermediate sector is perfectly competitive. The representative firm produces a homogeneous good, and sell it at its marginal cost. It uses only one input, labor, and all its production will be used to produce the final differentiated good.

Let $A$ be the production of the intermediate good, which is a function of the labor employed by the representative firm $L_A$ and its productivity $\frac{1}{\zeta}$ where $\zeta$ is the labor needed to produce one unit of intermediate good.
The production function for $A$ is:

$$ A(L_A) = \frac{1}{\zeta} L_A $$

As $A = \frac{1}{\zeta} L_A$, we have the total labor employed in the intermediate sector given by

$$ L_A = \zeta A $$

The production function leads to this profit function:

$$ \pi_A = P_A A - w \zeta A $$

Now, in perfect competition, the representative firm will sell its production at its marginal cost, so normalizing the common wage to 1 we have the price of the intermediate good:

$$ P_A = \zeta $$

### 2.2.2 Final good sector

There is a continuum of firms, each choosing to produce a different variety $\omega$. Production requires two factors, labor and the intermediate good. As the focus of this model is on agribusiness, it seems logical, and consistent with the literature on this sector (Requillart et al., 2000), to take these two inputs as complementary. Actually, a firm can’t substitute, for example, milk for workers in the production of cheese. So there exists a technological constraint in the production of the final good. For a given sector, each firm will use the same quantity of intermediate good to produce one unit of final good.

Nevertheless, a firm can be more efficient, and use a less labor-intensive technology to produce its variety. So, as in Melitz, the marginal productivity of labor differs across firms.

To enter the market, final sector firms have to pay a sunk entry cost equal to $f_e$ units of labor, but firms do not know their productivity prior to starting production. However, if firms are able to produce, they have to pay an overhead fixed cost $f_d$. Each firm produces a variety $\omega$ with a technology which leads to the production function:

$$ y_\omega(a, l) = \min\left(\frac{1}{\alpha} a_\omega; \frac{1}{\beta} l_\omega\right) $$

where:

- $y_\omega$ is the quantity of final good produced by the firm $\omega$;
- $a_\omega$ is the quantity of intermediate good used by the firm $\omega$;
- $\alpha$ is the quantity of intermediate good needed to produce one unit of final good (technological constraint, the same for all the sector);
- $l_\omega$ is the quantity of labor used by the firm $\omega$ for production;
- $\beta = \frac{1}{\varphi}$ is the quantity of labor used by the firm $\omega$ to produce one unit of final good. $\beta$ is the reciprocal of the firm’s labor-productivity $\varphi$. 


The total working population is divided between the intermediate and the final sector\footnote{This convenient assumption doesn’t change the results at this stage of the model.} so we can write $L = L_A + L_Y$. So we have the population available in the final sector given by:

$$L_Y = L - \zeta A$$

(8)

In monopolistic competition, each firm faces a residual demand curve with constant elasticity $\sigma$ and thus chooses a markup equal to $\frac{\sigma}{\sigma - 1} = 1/\rho$. This markup, with the marginal cost expressed in units of labor, leads to the pricing rule:

$$p(\omega) = \frac{1}{\rho}Cm = \frac{(\zeta\alpha + \beta)}{\rho}$$

(9)

Each firm produces its own variety, and each firm varies in terms of its labor-productivity level. Thus each labor-productivity level leads to a variety $\varphi$, and there exists a function $\omega (\varphi)$. Furthermore, in Melitz 2003, the firm’s productivity is defined as the inverse of its marginal cost, so let us defined the global-productivity level of the firm as the inverse of its marginal cost. We have then:

$$\psi(\varphi) = \frac{1}{Cm} = \frac{1}{\zeta\alpha + \beta} = \frac{\varphi}{1 + \zeta\alpha\varphi}$$

As for a given $\varphi$ we have one $\omega$ and one $\psi$, we can refer to a firm either by the variety produced ($\omega$), its labor-productivity ($\varphi$) or its global-productivity ($\psi$). So we can write the price rule as a function of the global-productivity level of the firm.

$$p(\omega(\psi)) = p(\psi) = \frac{1}{\rho\psi}$$

(10)

Focusing on global-productivity we find the same price as Melitz 2003. The main difference is that, in this paper, the productivity of a firm can be divided into a labor productivity level and an intermediate good productivity level, and only labor productivity vary across firms. We also have the same profit and revenue functions as in Melitz (2003):

$$\pi(\psi) = (1 - \rho)r(\psi) - f = \frac{r(\psi)}{\sigma} - f$$

(11)

$$r(\psi) = R \left[ \frac{p(\psi)}{P} \right]^{1-\sigma} = R [\rho \psi P]^{\sigma-1}$$

(12)

And we can write the ratios of any two firms’ outputs and revenues as a function of their global-productivity only.

$$\frac{y(\psi_1)}{y(\psi_2)} = \left[ \frac{\psi_1}{\psi_2} \right]^\sigma = \left[ \frac{\varphi_1 (1 + \zeta\alpha\varphi_2)}{\varphi_2 (1 + \zeta\alpha\varphi_1)} \right]^\sigma$$

(13)

$$\frac{r(\psi_1)}{r(\psi_2)} = \left[ \frac{\psi_1}{\psi_2} \right]^{\sigma-1} = \left[ \frac{(1 + \zeta\alpha\varphi_2) \varphi_1}{(1 + \zeta\alpha\varphi_1) \varphi_2} \right]^{\sigma-1}$$

(14)

In Melitz, these ratios only depend on the labor-productivity levels of firms\footnote{In Melitz we have $\frac{y(\psi_1)}{y(\psi_2)} = \left[ \frac{\varphi_1}{\varphi_2} \right]^\sigma$ and $\frac{r(\psi_1)}{r(\psi_2)} = \left[ \frac{\psi_1}{\psi_2} \right]^{\sigma-1}$.}. With the introduction of a second input, output and revenue ratios depend on global-productivity levels, thus intermediate good characteristics modify these ratios. We see that the more intensive in intermediate good
final good, or the more expensive the intermediate good, the lower the ratios are. The existence of an intermediate good under a fixed proportions technology reduces the advantage of more productive firms. This is an important result, because this predicts that in sectors more closely related to an intermediate sector, output and revenue differences will be lower, even if the heterogeneity of firms is not lower.

The results have implications regarding R&D. We find here that when there is an input which is used in a fixed proportion, this reduces the advantage of more productive firms. So if a firm invests R&D to improve its labor-productivity, its gain from this productivity improvement will be lower, and this reduces its incentive to invest. Thus we can expect that in this framework, R&D could be lower, because the intermediate good attenuates the impact of heterogeneity.

2.3 Convenient aggregation

An equilibrium will be characterized by a mass \( M \) of firms and a distribution \( \mu(\varphi) \) of labor-productivity levels over a subset of \([0, +\infty)\). We define \( \psi(\tilde{\varphi}) = \tilde{\psi} \) as the weighted harmonic average of firms’ global-productivity levels:

\[
\psi(\varphi) = \psi = \frac{1}{\varphi} + \frac{1}{\zeta} \alpha \varphi.
\]

\[
\tilde{\psi} = \left[ \int_0^\infty \psi^{1-\sigma} \mu(\varphi) \, d\varphi \right]^{\frac{1}{1-\sigma}}
\]  

(15)

\( \tilde{\psi} \) can also be written as a function of averages of firms labor-productivity levels and intermediate-productivity levels: \( \tilde{\psi} = \frac{\tilde{\varphi}}{1 + \zeta \alpha \tilde{\varphi}} \) with

\[
\tilde{\varphi}^{-1} = \int \varphi^{-1} \frac{y(\psi)}{y(\tilde{\psi})} \mu(\varphi) \, d\varphi
\]

\[
\tilde{\alpha} = \int \alpha \frac{y(\psi)}{y(\tilde{\psi})} \mu(\varphi) \, d\varphi
\]

As in Melitz, we can write all aggregate variables as functions of \( \tilde{\psi} \). So we have aggregated price index, aggregated revenue, aggregated output and aggregated profit written as:

\[
P = \left[ \int p(\psi)^{1-\sigma} M \mu(\varphi) \, d\varphi \right]^{\frac{1}{1-\sigma}} = M^{\frac{1}{1-\sigma}} p(\tilde{\psi})
\]

(18)

\[
R = \int r(\psi) M \mu(\varphi) \, d\varphi = M r(\tilde{\psi})
\]

(19)

\[
\Pi = \int \pi(\psi) M \mu(\varphi) \, d\varphi = M \pi(\tilde{\psi})
\]

(20)

\[
Y = M^{\frac{\pi+\gamma}{1-\sigma}} g(\tilde{\psi})
\]

(21)

\( \tilde{\psi} \) is also defined as the global-productivity level of a firm which has the average revenue among all firms:

\[
p(\tilde{\psi}) = \int p(\psi) y(\psi) \mu(\varphi) \, d\varphi = \int p(\psi) y(\psi) \mu(\varphi) \, d\varphi = \int \frac{1}{p(\psi) y(\psi)} g(\psi) \mu(\varphi) \, d\varphi
\]
2.4 Firm entry and exit

Potentials entrants are unlimited, prior to entry firms are identical. To enter the market, they have to pay a sunk cost $f_e$ measured in units of labor. Then the labor-productivity level $\varphi$ of each firm is randomly drawn from a common distribution $g(\varphi)$. $g(\varphi)$ is positive over $(0, \infty)$ and has a continuous cumulative function $G(\varphi)$.

Firms which have a low productivity level could decide not to produce and directly exit the market. If the firm does produce, it faces with probability $\delta$ a shock which forces it to exit. This probability is constant for each firm and at each period. As the productivity of a firm remains constant over time, its optimal profit level is constant too, until a shock forces it to exit. The value function of the firm is given by the actualized profit flows:

$$v(\psi) = \max \left\{ 0, \sum_{t=0}^{\infty} (1-\delta)^t \pi(\psi) \right\} = \max \left\{ 0, \frac{1}{\delta} \pi(\psi) \right\}$$  \hspace{1cm} (22)

If a firm has a global-productivity level too low, its actualized profit flow will be negative, so it will not enter the market. So there exists a threshold above which a firm can enter the market, and make profit. This threshold ($\psi^*$) is defined as the minimum value of $\psi$ that leads to non-negative firm value.

$$\psi^* = \inf \{ \psi : v(\psi) \geq 0 \}$$  \hspace{1cm} (23)

Given that $\psi = \frac{\varphi}{1 + \zeta \alpha \varphi}$ and that $\zeta$ and $\alpha$ are the same across all firm of a sector, $\psi^*$ is a function of $\varphi^*$, which is the labor-productivity threshold and can be written as:

$$\psi^* = \frac{\varphi^*}{1 + \zeta \alpha \varphi^*} = \inf \{ \varphi : v(\psi) \geq 0 \}$$

As in Melitz (2003), $\mu(\varphi)$ is the conditional distribution of $g(\varphi)$ on $[\varphi^*, +\infty[$

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi)} & \text{if } \varphi \geq \varphi^* \\ 0 & \text{if } \varphi < \varphi^* \end{cases}$$  \hspace{1cm} (24)

$p_e \equiv 1 - G(\varphi^*)$ is the ex-ante probability of successful entry. We now can write the aggregate productivity level as a function of the cutoff level $\varphi$.

$$\bar{\psi}(\varphi^*) = \bar{\psi} = \left[ \frac{1}{1-G(\varphi^*)} \int_{\varphi^*}^{\infty} \psi^{\sigma-1} g(\varphi) \, d\varphi \right]^{\frac{1}{\sigma-1}}$$  \hspace{1cm} (25)

2.5 Equilibrium conditions

As $\bar{\psi}$ is totally determined by the threshold $\varphi^*$, the average revenue and profit are also defined only by $\varphi^*$. Following Melitz’s model, we find a similar "zero profit condition" and "free entry condition", except that we have the global-productivity levels ($\psi$) instead of labor-productivity levels ($\varphi$).

Thus the zero profit condition is equal to :

$$\bar{\pi} = f \left( \left( \frac{\bar{\psi}}{\psi^*} \right)^{\sigma-1} - 1 \right) = f k(\psi^*)$$  \hspace{1cm} (ZPCd)
with \( k(\psi) = \left(\frac{\psi}{\bar{G}}\right)^{\sigma-1} - 1 \).

And the free entry condition:

\[
\tilde{\pi} = \frac{\delta f_e}{1 - G(\varphi^*)}
\]  

(FEC\textsubscript{d})

In addition to these conditions, at equilibrium, all factors of production must be used, so we must have an equilibrium between the production and the use of the intermediate good. As the use of the intermediate good is a function of the final good production, the equilibrium must give us a fixed relation between aggregate final good production and total intermediate good production.

We start from the equilibrium at the firm level

\[
y(\psi) = \frac{1}{\alpha} a(\psi) = \varphi_{l_p}(\psi)
\]

and we derive the equilibrium condition that relates the production of the intermediate good and the final good.

\[
A = \bar{\alpha} M y \left(\psi\right)
\]

(EC\textsubscript{d})

\[\text{2.6 Closed economy equilibrium}\]

Following Melitz, we use the free entry condition \(\text{FEC}\textsubscript{d}\) and the zero profit condition \(\text{ZPC}\textsubscript{d}\) to determine a unique  \(\varphi^*\), which determines the average profit level  \(\bar{\pi}\).

At the stationary equilibrium, all variables remain constant. The mass of new entrants must replace the mass of firms which exit \( (p_e M_e = \delta M) \); the use of labor for entry investment must be reflected in the amount of total labor available for final good production \( L_Y (L_Y = L_{ey} + L_{py}) \) where \( L_{ey} = M_c f_e \); aggregated payments for production (workers and intermediate good) must equal the difference between aggregate revenue and profit \( (L_{PY} + \zeta A = R - \Pi) \).

As in Melitz, the market clearing condition for labor used for entry leads to the fact that the total payment for entry labor matches the aggregate profit \( (L_{ey} = M_c f_e = \Pi) \).

And with the eq. \[\text{5}\] we can note

\[
R = L_{py} + L_{ey} + L_A = L
\]

The aggregate revenue is fixed exogenously by the size of the country.

At the equilibrium, all factors of production must be used. The equilibrium condition \(\text{EC}\textsubscript{d}\) gives us a relation between sectors. But as there exists a relation between labor used in final good sector and production of the final good, between production of the final good and production of the intermediate good, and between production of the intermediate good and labor used in the intermediate good sector, there exists a relation between labor used in both sectors.

\[
M y \left(\psi\right) = \bar{\varphi} (L_{PY} - M f) = \frac{L_A}{\zeta \bar{\alpha}}
\]

\[
\zeta \bar{\alpha} \bar{\varphi} (L_{PY} - M f) = L_A
\]

(27)

Moreover, there is a constraint on labor available, so that total labor is divided between final good production, final good investment, and intermediate good production. So we have:

\[
L_{py} + L_{ey} + L_A = L
\]

\[\text{See Appendix A for proof.}\]
This labor market equilibrium determines the mass of firms

\[ M = \frac{L}{\sigma (\bar{\pi} + f)}. \]  

(28)

Melitz uses another method to determine the mass of firms, and we find the same results. It seems logical that the use of labor in the intermediate sector affects the amount of labor available for the production of the final good. So we must have a decrease of the mass of firms in the final sector when the productivity of intermediate sector decreases (\( \zeta \) increase). As in eq.28 only \( \bar{\pi} \) is endogenous, we can suppose that the increase in labor use (\( \zeta \)) of the intermediate sector impacts positively the aggregate profit at the equilibrium level, even if this leads to an increase of the price of the intermediate good. Although we have proved that the equilibrium is unique and exists, we can’t really investigate the impact of \( \zeta \) on the aggregate profit level.

The mass of firms in the final sector, determines the price index and the other aggregate variables.

\[ P = M \frac{\pi}{\psi} \rho \left( \frac{\psi}{\bar{\pi}} \right) = \left( \frac{L}{(\bar{\pi} + f)\sigma} \right) \frac{1}{\rho} \left( \frac{\psi}{\bar{\pi}} \right) \]  

(29)

\[ R = M \pi (\tilde{\psi}) = \frac{L}{\sigma (\bar{\pi} + f)} \pi (\tilde{\psi}) \]  

(30)

\[ \Pi = M \pi (\tilde{\psi}) = \frac{L}{\sigma (\bar{\pi} + f)} \pi (\tilde{\psi}) \]  

(31)

\[ Y = M \frac{\pi}{\psi} y (\tilde{\psi}) = \left( \frac{L}{(\bar{\pi} + f)\sigma} \right) \frac{1}{\rho} \left( \frac{\psi}{\bar{\pi}} \right) y (\tilde{\psi}) \]  

(32)

Welfare per worker is given by relative revenue per worker: \( W = \frac{R}{P} \), and as \( R = L \), welfare per worker is defined as the opposite of price index:

\[ W = P^{-1} = M \frac{\pi}{\psi} \rho \left( \frac{1}{\frac{\psi}{\bar{\pi}} + \zeta} \right) \]  

(33)

Welfare per worker is an increasing function of the aggregate labor-productivity of the final sector, and it’s also an increasing function of the level of efficiency in the intermediate sector (opposite of intermediate good price). But its a decreasing function of the quantity of intermediate good needed to produce one unit of final good.

We have \( \bar{\alpha} = \alpha \int \left( \frac{\psi}{\bar{\pi}} \right)^{\sigma} M \mu (\varphi) d\varphi \). Thus the greater the heterogeneity of firms, the higher \( \int \left( \frac{\psi}{\bar{\pi}} \right)^{\sigma} M \mu (\varphi) d\varphi \), and the greater the negative impact of \( \alpha \) is. On the other hand, the higher the heterogeneity of firms, the greater the benefits from a gain in the use of the intermediate good is.

### 3 Open economy model

In this part, we will focus on the international strategy of firms of the final sector, and how this strategy could be affected by the presence of an intermediate good, and by its characteristics.

We assume that the world is composed of \( n+1 \) symmetric countries. The objective of this model is to give a theoretical framework to analyze the agribusiness sector, which is chosen because it has a close relation with an upstream sector. Furthermore, impending policy in the agricultural sector make it pertinent to focus on the agribusiness sector. To do so, we will make some assumptions
regarding the relationship between the intermediate and the final sector which will provide a way to represent the agricultural and the agribusiness one.

Our first assumption is that the intermediate good has a very high transport cost - which is the case for raw milk, or highly perishable products - which reduces the opportunity of international trade in this sector. In addition, protectionist and interventionist policies enacted by some countries or regional group can also limit the possibility of international trade for some agricultural products - like quotas in E.U. In this paper, we will opt for the extreme case, and we assume that the intermediate good is so expensive to transport that it is not traded internationally. The openness of borders will be the subject of an extension of this model.

Moreover, the assumptions of symmetric countries insure that the price of the intermediate good will be the same in each country, and that the consumption of this intermediate good by the final sector will also be the same. Adding the homogeneity of the intermediate good, there is no reason for international trade in this sector (if there are positive transport costs).

In our model, we assume that the intermediate good is not internationally traded. That is, it has an iceberg cost \( \tau_a \) which doesn’t allow for international trade. To ensure that assumption is true, we assume \( \tau_a \to +\infty \). Thus, in this model, firms need to use locally produced intermediate goods.

For the final good sector, assumptions and characteristics follow the Melitz model.

In the first section, we allow firms to serve foreign market only by exports, and in the second one, we allow them to choose between export and FDI.

### 3.1 Open economy model with exports only

In this section, firms from country \( i \) can choose to pay a fixed cost, \( f_x \), which represents the adaptation costs to international markets (distribution and servicing network), and an iceberg transport cost \( \tau_y = \tau > 1 \), to sell a part of their production in a foreign market, via exports.

If they do so, as they pay additional cost to serve foreign markets, they will sell their production at a higher price in foreign markets. Countries are symmetric, have the same size, the same distribution of productivity, so factors of production (labor and intermediate good) will have the same price.

The possibility to export doesn’t change the marginal production cost of the firm, so the domestic price will equal the closed economy price.

\[
p_d(\psi) = \frac{1}{p^\psi}.
\]

When a firm exports, it applies the same markup to its marginal cost as in closed economy, but due to the transport cost, its marginal cost increases, and the price of its product in the foreign markets is higher.

\[
p_x(\psi) = \frac{\tau}{p^\psi} = \tau p_d(\psi)
\]

As in a closed economy, we find the same results as Melitz, except that now the *global-productivity* level (\( \psi \)) replaces the *labor-productivity* level (\( \varphi \)) of Melitz.

We can decompose the firm revenue between what it earns from domestic sales and from exports. As countries are symmetric, if a firm exports to one country, it will export to all. So we have the
combined revenue of an exporter firm:

\[ r(\psi) = r_d(\psi) + nr_x(\psi) \]

And we have the domestic revenue which is the same as the revenue in closed economy \( r_d(\psi) = R[\rho \psi P]^\sigma \). Thus we have the revenue from export to one country:

\[ r_x(\psi) = \tau^{1-\sigma} r_d(\psi) \]

The combined revenue of a firm, \( r(\psi) \), depends on its export status.

\[
r(\psi) = \begin{cases} 
  r_d(\psi) & \text{if the firm does not export} \\
  r_d(\psi) + nr_x(\psi) & \text{if the firm exports to } n \text{ countries}
\end{cases}
\]

(34)

3.1.1 Firm entry, exit, and export status

As in Melitz, we assume that firms are indifferent between paying the export cost \( f_{ex} \) and paying the amortized per period portion of this cost \( f_x = \delta f_{ex} \) in every period. We separate what the firm will earn from domestic sales to what it will earn from exports.

Domestic profit:

\[ \pi_d(\varphi) = \frac{r_d(\psi)}{\sigma} - f \]

Single country export profit:

\[ \pi_x(\varphi) = \frac{r_x(\psi)}{\sigma} - f_x \]

(35)

A firm will export only if its export profit is positive \( \pi_x(\psi) \geq 0 \). Then, for a successful entrant, combined profit can be written as:

\[ \pi(\psi) = \pi_d(\psi) + \max \{0, n\pi_x(\psi)\} \]

Firm value is still given by

\[ v(\psi) = \max \left\{0, \frac{1}{\delta} \pi(\psi) \right\} \]

and the domestic cutoff level is defined as in the closed economy and represents the labor-productivity level below which a firm will not enter the market.

\[ \varphi^*_d = \inf \{ \varphi : v(\psi) > 0 \} \]

\[ \psi^*_d = \psi(\varphi^*_d) = \inf \{ \psi : v(\psi) > 0 \} \]

To this domestic cutoff level, we add the export cutoff level, which is the labor-productivity level below which a firm will not export:

\[ \varphi^*_x = \inf \{ \varphi > \varphi^*_d \text{ and } \pi_x > 0 \} \]

\[ \psi^*_x = \inf \{ \psi > \psi^*_d \text{ and } \pi_x > 0 \} \]

If these thresholds are equal, or if the export cutoff is below the domestic one, all successful
entrants will export. To have a coexistence of domestic and exporting firms, we must have \( \varphi^*_x > \varphi^*_d \).

\[
\varphi^*_x > \varphi^*_d \iff \tau^\sigma - 1 f_x > f
\]

To have a partitioning by export status, we assume that the structure of costs satisfies the inequality above. This condition is exactly the same as in Melitz, so the upstream characteristics do not affect the condition in which a separation occurs between domestic and export firms.

The successful entry probability doesn’t change in an open economy, so we still have \( p_x = 1 - G(\varphi^*_d) \) and a distribution \( \mu(\varphi) \) determined by the ex-ante distribution \( g(\varphi) \) conditional on successful entry: \( \mu(\varphi) = \frac{g(\varphi)}{1 - G(\varphi^*_d)} \). In addition, the probability that one of these successful entrants exports is given by \( p_x = \frac{1 - G(\varphi^*_d)}{1 - G(\varphi^*_x)} \). So there is a fraction \( p_x \) of firms that exports, hence a mass \( M_x = p_x M \) of exporting firms, and a mass \( M_t = M + nM_x \) of available varieties in each country (\( M \) domestic varieties and \( M_x \) varieties imported from each \( n \) other countries).

We set \( \eta(\varphi) \) as the conditional distribution of \( g(\varphi) \) on \([\varphi^*_x, +\infty[\) . It’s the distribution of exporting firms, and can be written as:

\[
\eta(\varphi) = \begin{cases} 
\frac{g(\varphi)}{1 - G(\varphi^*_x)} & \text{if } \varphi \geq \varphi^*_x \\
0 & \text{if } \varphi < \varphi^*_x 
\end{cases}
\]

### 3.1.2 Aggregation

We set \( \tilde{\psi}_x \) which is the global-productivity of the firm with the average revenue among exporting firms:

\[
\tilde{\psi}_x = \frac{1 - G(\varphi^*_d)}{1 - G(\varphi^*_x)} \int_{\varphi^*_x}^{\infty} \psi^{\sigma - 1} \mu(\varphi) \, d\varphi
\]

But as in Melitz, this aggregate productivity doesn’t reflect trade costs and fixed costs. Therefore we set \( \tilde{\psi}_t \) as the global-productivity which leads to the average revenue among all firms in a given market: domestic revenue \( r_d(\psi) \) for \( M \) firms with a labor-productivity \( \varphi \in [\varphi^*, \infty[ \) and export revenue \( r_x(\psi) \) on \( n \) export markets for \( M_x \) firms with a labor-productivity \( \varphi \in [\varphi^*_x, \infty[ \).

\[
\tilde{\psi}_t = \left\{ \frac{1}{M_t} \left[ M \tilde{\psi}_x^{\sigma - 1} + nM_x \left( \tau_x^{-1} \tilde{\psi}_x \right)^{\sigma - 1} \right] \right\}^{1/(\sigma - 1)}
\]

(36)

We also have a relation between the export global-productivity level \( \tilde{\psi}_x \) and the average labor productivity level \( \bar{\varphi}_x \) and intermediate-good productivity level \( \bar{\alpha}_x \) given by:

\[
\tilde{\psi}_x = \frac{\bar{\psi}_x}{1 + \zeta_x \bar{\varphi}_x}
\]

with

\[
\bar{\varphi}_x = \frac{1 - G(\varphi^*_d)}{1 - G(\varphi^*_x)} \int_{\varphi^*_x}^{\infty} \frac{y(\psi)}{\tilde{\psi}_x} \mu(\varphi) \, d\varphi
\]

(37)

\[
\bar{\alpha}_x = \frac{1 - G(\varphi^*_d)}{1 - G(\varphi^*_x)} \int_{\varphi^*_x}^{\infty} \varphi^{-1} \frac{y(\psi)}{\bar{\varphi}_x} \mu(\varphi) \, d\varphi.
\]

(38)
This allows us to write

\[
\tilde{\psi}_t = \left\{ \frac{1}{M_t} \left[ M \frac{\tilde{\varphi}}{1 + \zeta \tilde{\alpha} \varphi} \right]^{\sigma - 1} + n M_x \left( \frac{\tilde{\gamma}_t}{1 + \zeta \tilde{\alpha}_t \tilde{\varphi}_t} \right)^{\sigma - 1} \right\}^{\frac{1}{\sigma - 1}} \tag{39}
\]

A firm uses labor for its domestic production, without losing production, so it uses \( y(\psi) \) units of labor for this domestic production. Exporting firms produce for foreign markets more than will be consumed overseas, because of iceberg transport costs. For one unit consumed overseas, an exporting firm must produce \( \tau \) units of final good. So we have a local production \( y_x \) for an overseas consumption \( y_x \). Moreover, transport costs increase marginal costs of exporting firms, and thus the price level of imported goods. We have a price of imported goods defined as \( \tau \) times the price of domestic ones, and this reduces the demand for imported varieties. We have a demand for imported goods equals to \( \tau^{\sigma} \) times the demand for domestic varieties.

We thus have a domestic demand \( y(\psi) \), which is also the effective production for the local market, and a foreign demand \( \tau^{-\sigma} y(\psi) \) which leads to a local production for foreign markets equals to \( \tau \tau^{-\sigma} y(\psi) \). Thus, \( M \) domestic firms use \( y(\psi) \) + \( f \) units of labor to serve the domestic market, and \( M_x \) exporting firms use \( \frac{\tau^{1-\sigma} y(\psi)}{\varphi} + f_x \) units of labor to serve each of the \( n \) foreign markets.

We set \( \tilde{\varphi}_t \) as the labor-productivity level of the firm which leads the firm to employ the average number of workers among all firms in a market, and it’s also the harmonic weighted mean of productivity in the final good sector.

\[
M_t \frac{y(\tilde{\psi}_t)}{\tilde{\varphi}_t} + M_f + n M_x f_x = \int_{\varphi_0}^{\infty} \left( \frac{y(\psi)}{\varphi} + f \right) \mu(\varphi) d\varphi + n \int_{\varphi_0}^{\infty} \left( \frac{\tau^{1-\sigma} y(\psi)}{\varphi} + f_x \right) M_x \eta(\varphi) d\varphi
\]

As with labor, a firm uses the intermediate good for domestic production without attrition, so it uses \( \alpha y(\psi) \) units of final good. On the other foreign market, an exporting firm produces \( \tau^{1-\sigma} y(\psi) \). So exporting firms use \( \alpha \tau^{1-\sigma} y(\psi) \) units of intermediate good to serve each of the \( n \) foreign markets.

We set \( \tilde{\alpha} \) as the average intermediate productivity level which leads to an average consumption of the intermediate good.

\[
\tilde{\alpha}_t = \frac{1}{M_t} \left[ M \int_{\varphi_0}^{\infty} \alpha \frac{y(\psi)}{\tilde{\psi}_t} \mu(\varphi) d\varphi + M_x \frac{1 - G(\varphi^*_t)}{1 - G(\varphi^*_x)} n \tau^{1-\sigma} \int_{\varphi_0}^{\infty} \alpha \frac{y(\psi)}{\tilde{\psi}_t} \mu(\varphi) d\varphi \right] \tag{40}
\]

Then we can write \( \tilde{\psi}_t \) as a function of \( \tilde{\varphi}_t \) and \( \tilde{\alpha}_t \):

\[
\tilde{\psi}_t = \frac{\tilde{\varphi}_t}{1 + \zeta \tilde{\alpha}_t \tilde{\varphi}_t}
\]

The aggregate variable \( \tilde{\psi}_t \), as \( \tilde{\psi} \) in closed economy, plays an important role in the determination
of other aggregate variables:

\[ P = M_t \frac{1}{\rho \tilde{\psi}_t} \left( \tilde{\psi}_t \right) = M_t \frac{1 + \zeta \tilde{\psi}_t}{\rho \tilde{\psi}_t} \]  
\[ R = M_t \gamma d \left( \tilde{\psi}_t \right) \]  
\[ W = \frac{R}{L} M_t \frac{1}{\rho \tilde{\psi}_t} \]

and the average revenue of national firms is given by \( \bar{r} = \int_{\psi_d}^{\psi_u} r_d \left( \psi \right) \mu \left( \psi \right) d\psi + p_x \int_{\psi_u}^{\psi_x} \left( nr_x \left( \psi \right) \right) \eta \left( \psi \right) d\psi = \frac{R}{M} \) \( \bar{r} = r_d \left( \tilde{\psi}_t \right) + p_x nr_x \left( \tilde{\psi}_t \right) \) \( \bar{r} = r_d \left( \tilde{\psi}_t \right) + p_x nr_x \left( \tilde{\psi}_t \right) \)

as well as the average profit of national firms \( \bar{\pi} = \int_{\psi_d}^{\psi_u} \pi_d \left( \psi \right) \mu \left( \psi \right) d\psi + p_x \int_{\psi_u}^{\psi_x} \left( n \pi_x \left( \psi \right) \right) \eta \left( \psi \right) d\psi = \frac{\Pi}{M} \)

\[ \bar{\pi} = \pi_d \left( \tilde{\psi}_t \right) + np_x \pi_x \left( \tilde{\psi}_t \right) \]

### 3.1.3 Equilibrium conditions

We can write \( \psi_x^* \) as a function of \( \psi_d^* \), and \( \varphi_x^* \) as a function of \( \varphi_d^* \):

\[ \psi_x^* = \psi^* \left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} \]
\[ \varphi_x^* = \left[ \frac{1}{\varphi^*} \left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} + \zeta \alpha \left( \left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} - 1 \right) \right]^{-1} \]

Analyzing this relation between \( \varphi_x^* \) and \( \varphi_d^* \), we show that it’s dependant on the intermediate sector’s characteristics:

\[ \frac{\partial \varphi_x^*}{\partial \alpha} = \frac{\left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} \left( 1 - \left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} \right) \varphi_d^{2 \alpha}}{\varphi^* \left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} + \left( \left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} - 1 \right) \varphi_d^{2 \alpha}} > 0 \]

\[ \frac{\partial \varphi_x^*}{\partial \zeta} = \frac{\left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} \left( 1 - \left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} \right) \varphi_d^{2 \zeta}}{\left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} + \left( \left( \frac{f_x}{f_x} \right)^{\frac{1}{\sigma}} - 1 \right) \varphi_d^{2 \zeta}} > 0 \]

The more intensive in intermediate good the final good, or the more expansive the intermediate good, the greater the difference is between the domestic and the export threshold. For our application, this result says that for agrofood goods with a large agricultural component, exporting firms must be more productive to access foreign markets, even if the domestic threshold is not higher.

The price, or the reciprocal of intermediate sector efficiency, has the same effect. In countries

\( ^6 \) Not to be confused with \( r \left( \tilde{\psi}_t \right) \) which is the average level of revenue done by all firms present in a given market, and given by \( r \left( \tilde{\psi}_t \right) = \frac{R}{M} \).

\( ^7 \) Not to be confused with \( \pi \left( \tilde{\psi}_t \right) \) which is the average level of profit done by all firms present in a given market, and given by \( \pi \left( \tilde{\psi}_t \right) = \frac{\Pi}{M} \).
where the agricultural sector is less efficient, firms will suffer from this non-efficiency, and for the same domestic threshold, they will have to be more productive to export compared to firms from a country with a more efficient agricultural market.

As in a closed economy, we determine a zero profit condition $\text{ZPC}_x$ and a free entry condition $\text{FEC}_x$, which are two way to have the average profit as a function of the thresholds.

\[ \bar{\pi} = \pi_d (\tilde{\psi}) + np_x \pi_x (\tilde{\psi}_x) \]
\[ \bar{\pi} = f k (\psi^*_d) + p_x n f_x k (\psi^*_x) \]  
\hspace{1cm} (ZPC_x)

with $k (\psi) = \left[ \frac{\psi}{\delta} \right]^{\sigma - 1}$.

\[ \bar{\pi} = \frac{\delta f_x}{p_{in}} \]  
\hspace{1cm} (FEC_x)

As the export threshold is a function of the domestic threshold, $\text{ZPC}_x$ and $\text{FEC}_x$ give two different relations between the average profit and the domestic threshold. Following Melitz, these two relations will determine the threshold level and the average profit level.

As in a closed economy, we add to these two condition another one involving final good production and intermediate good supply. Starting with the equilibrium at the firm level

\[ y (\psi) = \frac{a (\psi)}{\alpha} = \varphi^l_p \]

we derive the equilibrium condition which is the relation between aggregate intermediate good production and aggregate final good production with exporting firms

\[ A = \tilde{\alpha}_t M_t y (\tilde{\psi}_t) \]  
\hspace{1cm} (EC_x)

3.1.4 Determination of the equilibrium

Following Melitz, we use the free entry condition and the zero profit condition to determine a unique $\varphi^*_d$, which determines $\varphi^*_x$ and $\bar{\pi}^*$. As in a closed economy, at the stationary equilibrium, all variables remain constant. The mass of new domestic entrants and new exporting entrants must replace the mass of domestic firms and exporting firms which exist ($p_c M_c = M$) ; the use of labor for entry investment must be reflected in the amount of total labor available for final good production ($L_Y = L_{eY} + L_{pY}$ where $L_{eY} = M_c f_c$) ; aggregated payments for production (workers and intermediate good) must equal the difference between aggregate revenue and aggregate profit ($L_{pY} + \zeta A = R - \Pi$).

We still have the total payment for entry labor which matches the aggregate profit $R = L_{pY} + L_{eY} + L_A = L$.

The equilibrium condition gives us a relation between intermediate good and final good production which can be expressed in terms of labor used. This leads to a relation between aggregate final good production, labor used in the final good sector, and labor used in the intermediate good sector.

*See Appendix B for proof.
In the final good sector, the amount of labor used by a firm depends on its export status:

$$l_{py}(\psi) = \begin{cases} l_p(\psi) + f = \frac{y(\psi)}{\varphi} + f \\ (l_p(\psi) + f) + n (l_{px}(\psi) + f_x) = (1 + n\tau^{1-\sigma}) \frac{y(\psi)}{\varphi} + f + nf_x \end{cases}$$

For domestic firms

For exporting firms

(51)

With the equation above and the equilibrium condition, we have the relation between aggregate final good production, labor used in final good sector, and labor used in intermediate good sector:

$$M_t \Psi (\psi_t) = \tilde{\varphi}_t (L_{py} - f M - nf_x M_x) = \frac{L_A}{\zeta \delta_t}.$$  

With the fact that the total amount of labor available is divided between the intermediate and the final good sector:

$$L = L_{py} + L_{cy} + L_A$$

we can derive the equilibrium mass of domestic firms in a country

$$M = \frac{L}{\sigma (\tilde{\varphi} + f + np_x f_x)}$$  

(52)

and the equilibrium mass of exporting firms ($M_x = p_x M$) and of available varieties on a market

$$M_t = (1 + np_x) \frac{L}{\sigma (\tilde{\varphi} + f + np_x f_x)}.$$  

Thus we can derive the equilibrium levels of aggregate variables:

$$P = \left( \frac{(1 + np_x) L}{\sigma (\tilde{\varphi} + f + np_x f_x)} \right)^{\frac{1}{1-\rho}} \frac{1}{\rho} \left( \frac{1}{\tilde{\varphi}_t} + \zeta \delta_t \right)$$

(53)

$$R = \frac{(1 + np_x) L}{\sigma (\tilde{\varphi} + f + np_x f_x)} r_d (\tilde{\psi}_t)$$

(54)

$$W = \frac{R}{L} M_t \Psi_t \rho \tilde{\psi}_t = \left( \frac{(1 + np_x) L}{\sigma (\tilde{\varphi} + f + np_x f_x)} \right)^{\frac{1}{1-\rho}} \frac{1}{\rho} \left( \frac{1}{\tilde{\varphi}_t} + \zeta \delta_t \right)$$

(55)

Intermediate sector characteristics impact aggregate variables in the same way than in a closed economy, particularly welfare per worker which is a decreasing function of the intensity of the linkage between the two sectors (price of the intermediate good and intensity of the final good in intermediate good).

### 3.2 Open economy model with export-FDI trade-off

In this section, we allow firms to serve foreign markets in a new way by creating foreign affiliates. Location of production doesn’t affect variety characteristics, each firm still produces only one variety, regardless of the country. We still assume symmetric countries, so when a firm has an affiliate, it has one in every foreign country, and as it produces the same variety in every plant, there will not be exports from affiliates. Thus export and horizontal foreign direct investment (HFDI) are alternative ways to serve foreign markets.

To create or buy a foreign affiliate, a firm must pay a fixed cost $f_I$. As in Helpman et al. (2004), this fixed cost includes the adaptation costs to foreign markets (distribution and servicing network) as was the case for $f_x$, as well as the cost of creating or acquiring an affiliate overseas...
and duplicating overhead production costs as was the case for \( f_d \). So we have \( f_I > f_x > f_d \).

As in Helpman \textit{et al.}, a firm will not invest abroad just because its profit doing from HFDI is positive. As there is a trade-off between export and HFDI, a firm will invest abroad when its investing profit is positive but also only if this investing profit is greater than its exporting profit.

In monopolistic competition, whatever the market, a firm sells its production with a markup \( \frac{1}{\rho} \) over its marginal cost, so we have these pricing rules:

\[
\begin{align*}
    p_d &= \frac{1}{\rho} C m_d & \text{For varieties produced by domestic firms} \\
    p_x &= \frac{1}{\rho} C m_x & \text{For imported varieties} \\
    p_I &= \frac{1}{\rho} C m_I & \text{For varieties produced by foreign affiliates}
\end{align*}
\]

We still have the same prices rules and profit function for domestic and exporting firms.

\[
\begin{align*}
    p_d(\psi) &= \frac{1}{\rho \psi} \\
    p_x(\psi) &= \frac{\tau}{\rho \psi}
\end{align*}
\]

To serve a foreign market by FDI, a firm has this cost function

\[
CT_I(\psi) = \left( \zeta \alpha + \frac{1}{\varphi} \right) y(\psi) + f_I
\]

which leads to the same price as domestic firms

\[
p_I = p_d = \frac{1}{\rho \psi}
\]

In other words, we have a price which depends on the location of production, but not on the nationality of firms.

### 3.2.1 Firm entry, exit, and international status

There are now three different types of firms: purely domestic firms which only produce for the domestic market, exporting firms which produce in the home country and export to all \( n \) foreign countries, and international firms which serve each of the \( n + 1 \) markets with a local plant (its headquarter in the home market and its foreign affiliate in a foreign market).

The combined profit of a firm, \( \pi(\psi) \), depends on its classification.

\[
\pi(\psi) = \begin{cases} 
    \pi_d(\psi) = \frac{r_d(\psi)}{\sigma} - f_d & \text{For domestic firms} \\
    \pi_d(\psi) + n \pi_x(\psi) = \frac{1 + n \pi_x}{\sigma} r_d(\psi) - f_d - nf_x & \text{For exporting firms} \\
    \pi_d(\psi) + n \pi I(\psi) = \frac{1 + n \pi I}{\sigma} r_d(\psi) - f_d - nf_I & \text{For multinational firms}
\end{cases}
\]

We still have the same definition of the domestic and the export thresholds:

\[
\begin{align*}
    \varphi^*_d &= \inf \{ \varphi : v(\psi) \geq 0 \} \\
    \psi^*_d &= \inf \{ \psi : v(\psi) \geq 0 \}
\end{align*}
\]
\[ \varphi^*_x = \inf \{ \varphi > \varphi^*_d \text{ et } \pi_x \geq 0 \} \]
\[ \psi^*_x = \inf \{ \psi > \psi^*_d \text{ et } \pi_x \geq 0 \} \]

to which we add the multinational threshold above which the profit to invest abroad is positive and greater than the profit to export:

\[ \varphi^*_f = \inf \{ \varphi \geq \varphi^*_x \text{ et } \pi_I \geq \pi_x \} \]
\[ \psi^*_f = \inf \{ \psi \geq \psi^*_x \text{ et } \pi_I \geq \pi_x \} \]

(60)

If \( \varphi^*_x = \varphi^*_f \) (thus \( \psi^*_x = \psi^*_f \)) all firms which can serve a foreign market will do it by HFDI, and there will not be exporting firms. To have a coexistence of exporting and multinational firms, we must have \( \varphi^*_f > \varphi^*_x \). To do so, we assume a cost structure as:

\[ f_I > \tau^{\sigma-1} f_x. \]

(61)

Then, if \( f_d < \tau^{\sigma-1} f_x < f_I \) there will be a range of threshold like \( \varphi^*_d < \varphi^*_x < \varphi^*_f \), and less productive firms will serve only domestic markets, more productive firms will serve foreign markets by exports, and the most productive firms will serve foreign markets by HFDI.

The combined revenue of a firm, \( r(\psi) \), depends on its type:

\[
    r(\psi) = \begin{cases} 
    r_d(\psi) & \text{For domestic firms} \\
    r_d(\psi) + nr_x(\psi) = (1 + n\tau^{1-\sigma}) r_d(\psi) & \text{For exporting firms} \\
    r_d(\psi) + nr_I(\psi) = (1 + n) r_d(\psi) & \text{For multinational firms}
    \end{cases}
\]

(62)

The probability of successful entry doesn’t change (\( p_e = 1 - G(\varphi^*_d) \)), and \( \mu(\varphi) \) is still the ex-ante distribution \( g(\varphi) \) conditional on successful entry: \( \mu(\varphi) = \frac{g(\varphi)}{1 - G(\varphi^*_d)} \). Moreover, we set \( \varepsilon(\varphi) \) as the conditional distribution of \( g(\varphi) \) on \( [\varphi^*_f; +\infty] \):

\[ \varepsilon(\varphi) = \begin{cases} 
    \frac{g(\varphi)}{1 - G(\varphi^*_f)} = \frac{\mu(\varphi)}{p_I} & \text{if } \varphi \geq \varphi^*_f \\
    0 & \text{if } \varphi < \varphi^*_f
    \end{cases} \]

(63)

The probability that a successful entrant invests abroad is equal to \( p_I = \frac{1 - G(\varphi^*_f)}{1 - G(\varphi^*_d)} \). \( p_x \) still represents the conditional probability that a successful entrant exports, but the fact that a portion of firms above the export threshold will serve foreign markets by FDI changes its definition. Now, exporting firms will be firms with a labor-productivity level between \( \varphi_x^* \) and \( \varphi^*_f \). So we have \( p_x = \frac{1 - G(\varphi^*_f) - [1 - G(\varphi^*_x)]}{1 - G(\varphi^*_d)} = G(\varphi^*_f) - G(\varphi^*_x) \).

We still have a probability \( p_x \) and a mass \( M_x = p_x M \) of exporting firms. Moreover, \( \eta(\varphi) \), which represents the ex-ante distribution \( g(\varphi) \) conditional on export status is also modified:

\[ \eta(\varphi) = \begin{cases} 
    0 & \text{if } \varphi \geq \varphi^*_f \\
    \frac{g(\varphi)}{G(\varphi^*_f) - G(\varphi^*_x)} = \frac{\mu(\varphi)}{p_x} & \text{if } \varphi_x^* \leq \varphi < \varphi^*_f \\
    0 & \text{if } \varphi < \varphi_x^*
    \end{cases} \]

(64)

In addition, there is a proportion \( p_I \) of firms which invests abroad, and thus a mass \( M_I = p_I M \) of multinational firms. Then, the total mass of available varieties in a country is given by \( M_t = \).
\[ M + nM_d + nM_I, \] and there is a mass \( M_H = M + nM_I \) of locally produced varieties.

### 3.2.2 Aggregation

Aggregate revenue in a country is composed of the revenue of domestic sales, the revenue of national exporting firms from their sales in \( n \) foreign countries, and the revenue of multinational firms coming from \( n \) foreign countries and operating in the home country.

\[
R = \int_{\bar{\varphi}_d}^{\infty} r_d (\psi) M_d (\varphi) \, d\varphi + \frac{n}{\bar{\varphi}_d} r_x (\psi) M_x \eta (\varphi) \, d\varphi + n \int_{\bar{\varphi}_d}^{\infty} r_I (\psi) M_I \varepsilon (\varphi) \, d\varphi
\]

We still have \( \bar{\psi}_x = \frac{\bar{\varphi}_x}{1 + \bar{\alpha}_x \bar{\varphi}_x} \) the global-productivity level of an exporting firm with average revenue:

\[
\bar{\psi}_x = \int_0^\infty r (\psi) \eta (\varphi) \, d\varphi
\]

\[
\bar{\psi}_x = \left[ \frac{1 - G (\varphi_I^d)}{G (\varphi_I^d) - G (\varphi_x^d)} \int_{\varphi_I^d}^{\infty} \psi^{\sigma - 1} \mu (\varphi) \, d\varphi \right]^{\frac{1}{\sigma - 1}}
\]

(65)

where

\[
\bar{\varphi}_x^{-1} = \int_0^\infty \varphi^{-1} \frac{y (\psi)}{y (\bar{\psi}_x)} \eta (\varphi) \, d\varphi = \frac{1 - G (\varphi_I^d)}{G (\varphi_I^d) - G (\varphi_x^d)} \int_{\varphi_I^d}^{\infty} \varphi^{-1} \frac{y (\psi)}{y (\bar{\psi}_x)} \mu (\varphi) \, d\varphi
\]

(66)

\[
\bar{\alpha}_x = \int_0^\infty \alpha \frac{y (\psi)}{y (\bar{\psi}_x)} \eta (\varphi) \, d\varphi = \frac{1 - G (\varphi_I^d)}{G (\varphi_I^d) - G (\varphi_x^d)} \int_{\varphi_I^d}^{\infty} \alpha \frac{y (\psi)}{y (\bar{\psi}_x)} \mu (\varphi) \, d\varphi.
\]

(67)

We set \( \bar{\psi}_I = \frac{\bar{\varphi}_I}{1 + \bar{\alpha}_I \bar{\varphi}_I} \) the global-productivity level of a multinational firm with average revenue:

\[
\bar{\psi}_I = \left[ \frac{1 - G (\varphi_I^d)}{1 - G (\varphi_I^d)} \int_{\varphi_I^d}^{\infty} \psi^{\sigma - 1} \mu (\varphi) \, d\varphi \right]^{\frac{1}{\sigma - 1}}
\]

(68)

where

\[
\bar{\varphi}_I^{-1} = \int_0^\infty \varphi^{-1} \frac{y (\psi)}{y (\bar{\psi}_I)} \varepsilon (\varphi) \, d\varphi = \frac{1 - G (\varphi_I^d)}{1 - G (\varphi_I^d)} \int_{\varphi_I^d}^{\infty} \varphi^{-1} \frac{y (\psi)}{y (\bar{\psi}_I)} \mu (\varphi) \, d\varphi
\]

(69)

\[
\bar{\alpha}_I = \int_0^\infty \alpha \frac{y (\psi)}{y (\bar{\psi}_I)} \varepsilon (\varphi) \, d\varphi = \frac{1 - G (\varphi_I^d)}{1 - G (\varphi_I^d)} \int_{\varphi_I^d}^{\infty} \alpha \frac{y (\psi)}{y (\bar{\psi}_I)} \mu (\varphi) \, d\varphi.
\]

(70)

And \( \bar{\psi}_d \), the aggregate global productivity level of domestic firms remains unchanged

\[
\bar{\psi}_d = \left[ \int_{\varphi_d}^{\infty} \psi^{\sigma - 1} \mu (\varphi) \, d\varphi \right]^{\frac{1}{\sigma - 1}} = \frac{\bar{\varphi}_d}{1 + \bar{\alpha}_d \bar{\varphi}_d}
\]
Thus we can write 

\[ \tilde{\varphi}_d^{-1} = \int_{\varphi_d^z}^{\infty} \varphi^{-1} \frac{y(\varphi)}{y(\tilde{\varphi}_d)} \mu(\varphi) \, d\varphi \]

\[ \tilde{\alpha}_d = \int_{\varphi_d^z}^{\infty} \alpha \frac{y(\varphi)}{y(\tilde{\varphi}_d)} \mu(\varphi) \, d\varphi. \]

We set \( \tilde{\psi}_t \) as the new aggregate global-productivity level of all firms on a market, considering trade costs for exporting firms and foreign affiliates sales, which is given by the average revenue for all firms selling in a market:

\[ M_t r_t \left( \tilde{\psi}_t \right) = \int_{\varphi_d^z}^{\infty} r_d(\varphi) M(\varphi) d\varphi + \frac{1}{p_x} \int_{\varphi_d^z}^{\infty} m_r(\varphi) M_x(\varphi) d\varphi + \frac{1}{T_x} \int_{\varphi_d^z}^{\infty} r_t(\varphi) M(\varphi) d\varphi \]

\[ \tilde{\psi}_t = \left( \frac{1}{M_t} \left[ M_t \tilde{\varphi}_d^{-1} + M_x n(1-\sigma) \tilde{\psi}_x^{-1} + M_I n \tilde{\varphi}_I^{-1} \right] \right)^{\frac{1}{1-\sigma}} \]  

(71)

Setting \( \tilde{\varphi}_t \) the average labor productivity level of all firms

\[ \tilde{\varphi}_t^{-1} = \frac{1}{M_t} \left[ \int_{\varphi_d^z}^{\infty} \varphi \frac{y_d(\varphi)}{y(\tilde{\varphi}_t)} \mu(\varphi) \, d\varphi + M_x n(1-\sigma) \tilde{\psi}_x^{-1} \frac{1 - G(\varphi_d^z)}{G(\varphi_d^z)} \int_{\varphi_d^z}^{\infty} \varphi \frac{y_d(\varphi)}{y(\tilde{\varphi}_t)} \mu(\varphi) \, d\varphi \right] \]

(72)

\[ + M_I n \frac{1 - G(\varphi_d^z)}{G(\varphi_d^z)} \int_{\varphi_d^z}^{\infty} \varphi \frac{y_d(\varphi)}{y(\tilde{\varphi}_t)} \mu(\varphi) \, d\varphi \]

(73)

and \( \tilde{\alpha}_t \) the average intermediate intensity level

\[ \tilde{\varphi}_t^{-1} = \frac{1}{M_t} \left[ \int_{\varphi_d^z}^{\infty} \varphi \frac{y_d(\varphi)}{y(\tilde{\varphi}_t)} \mu(\varphi) \, d\varphi + M_x n(1-\sigma) \tilde{\psi}_x^{-1} \frac{1 - G(\varphi_d^z)}{G(\varphi_d^z)} \int_{\varphi_d^z}^{\infty} \varphi \frac{y_d(\varphi)}{y(\tilde{\varphi}_t)} \mu(\varphi) \, d\varphi \right] \]

\[ + M_I n \frac{1 - G(\varphi_d^z)}{G(\varphi_d^z)} \int_{\varphi_d^z}^{\infty} \varphi \frac{y_d(\varphi)}{y(\tilde{\varphi}_t)} \mu(\varphi) \, d\varphi \]

(73)

we can write

\[ \tilde{\psi}_t = \frac{\tilde{\varphi}_t}{1 + \zeta \tilde{\alpha}_t \tilde{\varphi}_t}. \]

Thus \( \tilde{\psi}_t \) determines other aggregate variables:

\[ P = M_t \tilde{\varphi}_t \frac{1}{1 + \zeta \tilde{\alpha}_t} \]

(74)

\[ R = M_t r_d \left( \tilde{\psi}_t \right) \]

(75)

\[ W = \frac{R}{L} P^{-1} = M_t \rho \frac{\rho}{(1 + \zeta \tilde{\alpha}_t)} \]

(76)

The average revenue of a domestic firm equals the average domestic revenue, plus the average export revenue if it exports, plus the average investing revenue if it invests abroad. A successful entrant export with a probability \( p_x \), and invests abroad with a probability \( p_I \). Moreover, given the
symmetric assumption, the revenue of foreign affiliates in a country equals the revenue of affiliates abroad, so the average revenue is the aggregate revenue divided by the mass of domestic firms. Thus we have:

\[ \bar{r} = \frac{R}{M} = r_d \left( \bar{\psi}_d \right) + np_x r_x \left( \bar{\psi}_x \right) + np_I r_I \left( \bar{\psi}_I \right) \]

By the same token we can derive the average profit level of national firms:

\[ \bar{\Pi} = \frac{\Pi}{M} = \pi_d \left( \bar{\psi}_d \right) + np_x \pi_x \left( \bar{\psi}_x \right) + np_I \pi_I \left( \bar{\psi}_I \right) \]

3.2.3 Equilibrium conditions

The investing threshold represents the \textit{labor-productivity} level for which a firm has the same profit from exporting or investing abroad.

We can write \( \psi^*_I \) as an increasing function of \( \psi^*_d \), and consequently \( \varphi^*_I \) as a increasing function of \( \varphi^*_d \).

\[ \psi^*_I = \psi^*_d \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} \] \hspace{1cm} (77)

\[ \varphi^*_I = \left( \frac{1}{\varphi^*_d} + \zeta \alpha \right) \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} - \zeta \alpha \right]^{-1} \] \hspace{1cm} (78)

As for the export threshold, we show that the relation between the domestic threshold and the investing threshold depends on the intermediate sector’s characteristics:

\[ \frac{\partial \varphi^*_I}{\partial \alpha} = \frac{\varphi^* \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} \left( 1 - \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} \right) \varphi^*_d^{2} \zeta}{\left( \varphi^* \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} + \left( \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} - 1 \right) \varphi^*_d^{2} \zeta \alpha \right)^{2}} > 0 \] \hspace{1cm} (79)

\[ \frac{\partial \varphi^*_I}{\partial \zeta} = \frac{\varphi^* \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} \left( 1 - \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} \right) \varphi^*_d^{2} \alpha}{\left( \varphi^* \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} + \left( \left[ \frac{f_I - f_x}{f(I - \tau^{1 - \sigma})} \right]^{\frac{1}{\varphi^*_d}} - 1 \right) \varphi^*_d^{2} \zeta \alpha \right)^{2}} > 0 \] \hspace{1cm} (80)

The more intensive in intermediate good the final good, or the more expansive the intermediate good, the greater is the difference between the domestic and the investing threshold. For our application, this result says that for agrofood goods with a large agricultural component, firms must be more productive to access foreign markets through horizontal foreign direct investment, even if the domestic threshold is not higher than sector with a smaller agricultural component. The price, or the reciprocal of intermediate sector efficiency, has the same effect. In countries where the agricultural sector is less efficient, firms will suffer from this non-efficiency, and for the same domestic threshold, they will have to be more productive to invest abroad compared to firms from a country with a more efficient agricultural market.

As in previous frameworks, we determine a zero profit condition (ZPC), and a free entry condition (FEC), which are two ways to express average profit as a function of the thresholds:

\[ \bar{\Pi} = \pi_d \left( \bar{\psi}_d \right) + np_x \pi_x \left( \bar{\psi}_x \right) + np_I \pi_I \left( \bar{\psi}_I \right) \]

\[ \bar{\Pi} = \frac{\Pi}{M} = \pi_d \left( \bar{\psi}_d \right) + np_x \pi_x \left( \bar{\psi}_x \right) + np_I \pi_I \left( \bar{\psi}_I \right) \]  \hspace{1cm} (ZPC)

22
where \( k(\psi) = \left[ \frac{\psi}{\bar{\psi}} \right]^{\sigma - 1} \) and \( \kappa(\psi^*_I) = \left[ \frac{\psi}{\bar{\psi}} \right]^{\sigma - 1} \frac{(1 - \frac{\delta}{\kappa})}{(1 - \gamma)} - 1 \)

\[
\bar{\pi} = \frac{\delta f_e}{1 - G(\varphi^*_d)}.
\]

As all thresholds could be expressed as functions of the domestic one, the zero profit conditions and the free entry condition give two different relations between the domestic threshold and the average profit level. As in the previous frameworks, we add another condition involving final good production and intermediate good supply. Starting with the equilibrium at the firm level

\[
y(\psi) = \frac{a(\psi)}{\alpha} = \varphi l_p
\]

we derive the equilibrium condition between sectors

\[
M_t y(\psi_t) = \frac{A}{\alpha_t}
\]

3.2.4 Determination of the equilibrium

We still use the free entry condition and the zero profit condition to determine a unique \( \varphi^*_d \), which determines \( \varphi^*_x \), \( \varphi^*_I \) and \( \bar{\pi}^* \).

We have the same variables that remain constant as in previous models, which leads to the same equalities: the mass of new entrants must replace the mass of firms and which exit \( (p_e M_e = M) \); the use of labor for entry investment must be reflected in the amount of total labor available for final good production \( (L_Y = L_e Y + L_p Y) \) where \( L_{eg} = M_e f_e \); aggregated payments for production must equal the difference between aggregate revenue and aggregate profit \( (L_p Y + \zeta A = R - I) \) and the total payment for entry labor matches the aggregate profit \( (R = L_p Y + L_e Y + L_A = L) \).

The relation given by the equilibrium condition \( \text{EC}_I \) can be expressed in terms of labor used by each sector. In the final good sector, the amount of labor used by a firms depends on its international status:

\[
l_{p_y}(\psi) = \begin{cases} 
  l_p(\psi) + f = \frac{y(\psi)}{\varphi} + f & \text{For domestic firms} \\
  (l_p(\psi) + f) + n (l_{px}(\psi) + f_x) = (1 + n\tau^{1-\sigma}) \frac{y(\psi)}{\varphi} + f + nf_x & \text{For exporting firms} \\
  (l_p(\psi) + f) + n (l_{pI}(\psi) + f_I) = (1 + n) \frac{y(\psi)}{\varphi} + f + nf_I & \text{For multinational firms}
\end{cases}
\]

From this equation and the equilibrium conditions, we can derive the relation between final good production, labor used in the final good sector, and labor used in the intermediate good sector.

\[
M_t y(\psi_t) = \tilde{\varphi}_I (L_p Y - M f_d - nM_x f_x - nM_I f_I) = \frac{L_A}{\zeta \alpha_I}
\]

And as the total amount of labor available is divided between the two sectors \( (L = L_p Y + L_e Y + L_A) \), we can derive the mass of domestic firms at the equilibrium in each country:

\[
M = \frac{L}{\sigma (\bar{\pi} + f_d + nf_x + nf_I)}
\]

\(^9\)See Appendix C for proof.
which determines the total mass of varieties available in a country

$$M_t = M + nM_x + nM_I = \frac{L[1+n(p_x+p_I)]}{\sigma(\pi + f_d + np_x f_x + np_I f_I)}$$

and all aggregate variables

$$P = \left( \frac{L[1+n(p_x+p_I)]}{\sigma(\pi + f_d + np_x f_x + np_I f_I)} \right)^{\frac{\rho}{\rho-1}} \left( \frac{1}{\hat{\phi}_t} + \zeta \hat{\alpha}_t \right)$$  \hspace{1cm} (82)

$$R = \left( \frac{L[1+n(p_x+p_I)]}{\sigma(\pi + f_d + np_x f_x + np_I f_I)} \right)^{\frac{\rho}{\rho-1}} r_d \left( \tilde{\psi}_t \right)$$  \hspace{1cm} (83)

$$W = \frac{R}{L} P^{-1} = \left( \frac{L[1+n(p_x+p_I)]}{\sigma(\pi + f_d + np_x f_x + np_I f_I)} \right)^{\frac{\rho}{\rho-1}} \frac{\rho}{\left( \frac{1}{\hat{\phi}_t} + \zeta \hat{\alpha}_t \right)}$$  \hspace{1cm} (84)

Intermediate sector characteristics still impact aggregate variables in the same way as in a closed economy and as in an open economy without FDI. Welfare per worker remains a decreasing function of the intensity of the linkage between two sectors, which can be measured as the intensity of the final good in intermediate good and the price of the final intermediate good.

4 Conclusion

This paper has described an extension of the Melitz core model of heterogeneous firms with intermediate goods. This paper investigates how upstream sector characteristics and the production process of the downstream sector affect the impact of heterogeneity of firms. In this paper, we show that the characteristics of intermediate goods can shape the international strategy of firms aside from any consideration of comparative advantage. Indeed, as firms are assumed to use one input heterogeneously and one input homogeneously, the greater the use of either input in the final good, or the higher its price, the greater the impact of this input. In this model, firms are heterogeneous with respect to labor use, and homogeneous in term of the intermediate good. We show that the greater the use of the intermediate good in the final good, and the higher the price of the intermediate good, the more attenuated is the heterogeneity of firms. Thus, firms will need a higher labor productivity to export or make FDI. In addition, relations between productivity thresholds above which a firm can produce, export, or invest abroad, are affected by the intermediate good characteristics. The stronger the linkage between sectors (intensity in intermediate good and price of the latter), the higher the gap between thresholds.

As we extend the Melitz model, dividing global productivity into two parts, we can easily extend this model to more inputs. Considering that in this model the intermediate good is, in fact, a composite of all inputs used in fixed proportion, and that labor is, in fact, a composite of all heterogeneous inputs. Then the price of the intermediate good and of labor are also average prices of inputs. In this case, results are similar, and the impact of each input will depend on its relative weight. Fixed proportion inputs and heterogeneous inputs affect the heterogeneity of firms in contrary ways.

There exist two main limitations to this model. First, we can’t analyze the direct effect of the intermediate good on the domestic threshold and on average profit because, as in Melitz (2003), we don’t have an explicit expression. The second limitation concerns hypotheses. This model is too simplistic and doesn’t allow us to make predictions or policy recommendations. However, by
adding appropriate extensions, this model can be used as a basis to analyze some sectors which are closely related to others, like agribusiness is with agriculture. One of the main goals of this paper was to analyze the theoretical effects of a liberalization of the agricultural sector on international agribusiness activities. To do so, the next step is to introduce asymmetric countries and to allow international trade in the intermediate sector to show what happens to the international position of the final-good sector firms if we reduce exchange costs in the intermediate good sector.
References


Appendices

A. Existence uniqueness of the equilibrium in a closed economy

We start from the zero profit condition and the free entry condition:

\[
\dot{\pi} = \frac{\delta f_e}{1 - G(\varphi^*)} \\
\ddot{\pi} = f k(\psi^*)
\]

where \(k(\psi^*) = \left(\frac{\dot{\psi}}{\psi^*}\right)^{\sigma-1} - 1\)

Then

\[
\frac{\delta f_e}{1 - G(\varphi^*)} = f k(\varphi^*) \\
k(\psi^*) \left[1 - G(\varphi^*)\right] = \frac{\delta f_e}{f}.
\]

As \(\frac{\delta f_e}{f}\) is constant and exogenous, showing that the curve \(k(\psi^*) \left[1 - G(\varphi^*)\right]\) cut the curve \(\frac{\delta f_e}{f}\) ensure that the equilibrium exists. Following is a proof that \(k(\psi^*) \left[1 - G(\varphi^*)\right]\) is a decreasing function from infinity to zero on \([0, \infty]\).

We define \(f(\varphi^*) = k(\psi^*) \left[1 - G(\varphi^*)\right]\). The derivative

\[
\frac{\partial f(\varphi^*)}{\partial \varphi^*} = \frac{(1 - \sigma)}{\varphi^* (1 + \zeta \alpha \varphi^*)} \left[1 - G(\varphi^*)\right] [k(\psi^*) + 1] > 0
\]

shows that \(f(\varphi^*)\) is a decreasing function of \(\varphi^*\).

Moreover these limits

\[
\begin{align*}
\lim_{\varphi^* \to 0^+} f(\varphi^*) &= +\infty \\
\lim_{\varphi^* \to +\infty} f(\varphi^*) &= 0
\end{align*}
\]

prove that \(f(\varphi^*) = k(\psi^*) \left[1 - G(\varphi^*)\right]\) cut the curve \(\frac{\delta f_e}{f}\) (existence of the equilibrium) only one time on the interval (uniqueness).

B. Existence uniqueness of the equilibrium in open economy with exports only

We use the zero profit condition \(\text{(ZPC}_x\text{)}\) and the free entry condition \(\text{(FEC}_x\text{)}\):

\[
\dot{\pi} = f k(\psi^*_d) + p_x n f_x k(\psi^*_x) \\
\ddot{\pi} = \frac{\delta f_e}{p_{in}}
\]

Where \(k(\psi^*_d) = \left(\frac{\psi(\varphi^*_d)}{\psi^*_d}\right)^{\sigma-1} - 1\) and \(k(\psi^*_x) = \left(\frac{\psi(\varphi^*_x)}{\psi^*_x}\right)^{\sigma-1} - 1\).
Then we look for the \( \varphi^*_d \) which respect this condition:

\[
\frac{\delta f_e}{1 - G(\varphi^*_d)} = f k(\psi^*_d) + p_x n f_x k(\psi^*_x)
\]

\[
f(1 - G(\varphi^*_d)) \left( \frac{\bar{\psi}(\varphi^*_d)}{\psi^*_d} \right)^{\sigma - 1} - 1 \right) + n f_x \left[ (1 - G(\varphi^*_d)) \left( \frac{\bar{\psi}(\varphi^*_d)}{\psi^*_x} \right)^{\sigma - 1} - 1 \right] = \delta f_e.
\]

We define \( j(\varphi) = k(\psi) [1 - G(\varphi)] \)

\[
fj(\varphi^*_d) + n f_x j(\varphi^*_x) = \delta f_e
\]

We show that \( j(\varphi^*) \) was a decreasing function of \( \varphi^* \) from infinity to zero on \([0, \infty[\) and \( \varphi^*_x \) is an increasing function of \( \varphi^*_d \), then \( j(\varphi^*_d) \) is a decreasing function of \( \varphi^*_d \), and \( f j(\varphi^*_d) + n f_x j(\varphi^*_x) \) is also a decreasing function of \( \varphi^*_d \). Moreover, given the structure of costs, we have:

\[
\lim_{\varphi^*_d \to +\infty} fj(\varphi^*_d) + n f_x j(\varphi^*_x) = 0
\]

\[
\lim_{\varphi^*_d \to 0} fj(\varphi^*_d) + n f_x j(\varphi^*_x) = +\infty
\]

Then the curve \( fj(\varphi^*_d) + n f_x j(\varphi^*_x) \) cut only one time the curve \( \delta f_e \), which insure the existence and uniqueness of the equilibrium cut-off \( \varphi^* \). This cut-off determines \( \varphi^*_x \), other aggregate variables \( \bar{\psi}, \bar{\psi}_x \) and \( \bar{\psi}_t \), and the probabilities \( p_{in} \) and \( p_x \).

C. Existence uniqueness of the equilibrium in open economy with horizontal FDI

We use the free entry condition and the zero profit condition:

\[
\bar{\pi} = \frac{\delta f_e}{1 - G(\varphi^*_d)}
\]

\[
\bar{\pi} = f k(\psi^*) + p_x n f_x k(\psi^*_x) + n p_{in} f_{I1} k(\psi^*_I)
\]

where \( k(\psi) = \left( \frac{\bar{\psi}}{\psi} \right)^{\sigma - 1} - 1 \) and \( \kappa(\psi^*_I) = \left( \frac{\bar{\psi}_I}{\psi^*_I} \right)^{\sigma - 1} \frac{1 - \bar{\psi}_I}{1 - \psi^*_I} - 1 \)

This leads to

\[
f(1 - G(\varphi^*_d)) k(\psi^*) + n f_x k(\psi^*_x) [G(\varphi^*_I) - G(\varphi^*_d)] + (1 - G(\varphi^*_I)) n f_{I1} k(\psi^*_I) = \delta f_e.
\]

As for the previous equilibriums, to insure the existence and uniqueness of the equilibrium, the left side must cut the right side, but just one time.

To know the variation of \( f(1 - G(\varphi^*_d)) k(\psi^*) + n f_x [1 - G(\varphi^*_d)] k(\psi^*_x) + (1 - G(\varphi^*_I)) [n f_{I1} k(\psi^*_I) - n f_x k(\psi^*_x)] \) we need to assume that \( g(\varphi) \) is a decreasing function of \( \varphi \) on \([\varphi^*_d, \varphi^*_I] \). Given this assumption, \( f(1 - G(\varphi^*_d)) k(\psi^*) + n f_x [1 - G(\varphi^*_d)] k(\psi^*_x) + (1 - G(\varphi^*_I)) [n f_{I1} k(\psi^*_I) - n f_x k(\psi^*_x)] \) is a decreasing function of \( \varphi^*_d \). So if the equilibrium exists, it’s unique.
Moreover, by using L’Hopital rule, we find these limits:

\[
\lim_{\varphi_2 \to 0} f (1 - G (\varphi_2^*)) k (\psi^*) + n f_x k (\psi^*) [G (\varphi_1^*) - G (\varphi_2^*)] + (1 - G (\varphi_1^*)) n f_I k (\psi_1^*) = 0
\]

\[
\lim_{\varphi_2 \to +\infty} f (1 - G (\varphi_2^*)) k (\psi^*) + n f_x k (\psi^*) [G (\varphi_1^*) - G (\varphi_2^*)] + (1 - G (\varphi_1^*)) n f_I k (\psi_1^*) = 0
\]

which insure the existence of the equilibrium.