Trade and Growth: The Impact of Selection and Imitation

Sarah Stölting
European University Institute
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-Very Preliminary Version-

Abstract

This paper develops an endogenous growth and trade model with heterogeneous firms to analyze the impact of intra-industry trade on productivity growth. Growth is generated through a mechanism of selection, and sustained by entrants imitating successful incumbents. Firms are subject to idiosyncratic productivity shocks, forcing some firms to exit, resulting in an increase in the average productivity of the economy. The intra-industry effect of trade works through self-selection of the most productive firms into the export market, and leads to a reallocation of resources towards more efficient firms. Since the effect of selection and imitation on growth is amplified by the trade-induced selection process, opening up the economy to trade increases the growth rate of productivity.

Keywords: Endogenous growth, Intra-industry trade, Heterogeneous firms, Selection

JEL-Codes: F10, L11, O40

1 Introduction

This paper analyzes the impact on productivity growth of opening up an economy to costly trade. For this purpose an endogenous growth model with heterogeneous firms and intra industry-trade is developed. Both the effects of trade and growth on the economy are driven by a mechanism of selection, and growth is sustained by entrants imitating successful incumbents.

In recent years there has been an increasing number of empirical and theoretical research papers analyzing the effects of trade on productivity. Bernard and Jensen (1995) published one of the first
papers using firm-level data to investigate productivity differences between exporting and non-exporting firms. Since then, there has been a number of papers based on firm-level data from different countries. The two most important results of these studies are the following: First, there are large differences within industries in the export behavior of firms. Even in the so-called export-sectors, a large part of firms sell their products only in the domestic market. Secondly, exporting firms have higher performance characteristics than non-exporting firms, i.e. their productivity tend to be significantly higher, they are larger, more capital intensive and pay higher wages. Bernard and Jensen (1995) find that labor productivity for exporters is approximately a third greater than for non-exporters in the US in 1987. Concerning total factor productivity, Bernard et al. (2007) show that exporters are more productive by 3%. Their study is based on US data from the year 2000. The question of causality, i.e. whether more efficient firms become exporters or whether firms improve their performance after entering the export market, has been addressed by Bernard and Jensen (1999), who are also analyzing US data. They find clear evidence for more efficient firms becoming exporters, since performance measures are higher ex-ante for exporters. These differences, related to the export status among firms within industries, suggest that there is a self-selection of more productive firms into export markets. Similar evidence exists for different countries over different time periods (e.g. Baldwin and Gu (2004) for Canada, Eaton et al. (2004) for France and Van Biesebroeck (2005) for selected Sub-Saharan countries, among others).\footnote{Wagner (2007) provides a detailed overview of existing studies on productivity characteristics of exporting firms.}

Both 'old' trade theory and 'new' trade theory fail to consider firm level differences within sectors. New models have been developed in the last years in order to take into account intra-industry heterogeneity in terms of productivity. Important contributions are the models developed by Melitz (2003), Bernard et al. (2003) and Eaton and Kortum (2002). The focus here will be on Melitz (2003), which is a combination of the trade model of Krugman (1980) and the dynamic industry model of Hopenhayn (1992). As in Krugman (1980) the underlying assumptions of the model are CES preferences, monopolistic competition, increasing returns to scale and variable iceberg-type costs to trade. Melitz (2003) introduces some additional assumptions on heterogeneity of firms and on trade barriers: firms have different levels of labor productivity, the productivity of each firm is drawn randomly and firms face fixed costs of trade when exporting. This departure from the Krugman (1980) model yields the following result: exposing a country to costly trade makes only the more productive firms being involved in export activities, i.e. their profits and their market share increase, and forces the least productive ones to exit the market. This means that opening up to costly trade leads to an increase in productivity by reallocating resources to more efficient firms, i.e. through a mechanism of selection.

An important issue missing in trade models with intra-industry heterogeneity is productivity growth over time. The approach in the Melitz (2003) model assumes zero-growth in the steady state.
There have been very few papers which introduce growth in this framework, among them Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2006). In both papers endogenous growth comes from innovation of new product varieties, but there are differences in the assumptions concerning R&D. The focus of the analysis is on trade liberalization, while Melitz (2003) explores mainly the effect on productivity when a closed economy opens up to costly trade. Baldwin and Robert-Nicoud (2008) find that openness can either lead to slower or faster growth, depending on the impact of a reduction in trade costs on marginal costs of innovation in different R&D specifications. The main result of Gustafsson and Segerstrom (2006) is dependent on the size of intertemporal knowledge spillovers in R&D. Trade liberalization with weak spillovers leads to an increase in productivity growth, and with strong spillovers to a decrease productivity growth. In contrast to Baldwin and Robert-Nicoud (2008), the effect on the productivity growth rate is only temporary.

The ambiguous result of both papers is in line with the empirical evidence on the effect of trade on growth. Lopez (2005) and Berg and Krueger (2003) provide surveys on empirical studies analyzing whether trade has a positive impact on the growth rate of the economy, and they show that there is a large divergence in the evidence. While some papers find that the relationship is positive (mostly without being able to establish causality due to endogeneity problems), other papers find no significant correlation. On the other hand, as mentioned above, there is very clear and strong evidence for self-selection of highly productive firms into the export market. This mechanism of self-selection leads to a reallocation of resources from low-productivity to high-productivity firms. The reallocation of resources can be of great importance to the evolution of productivity growth. For example, Pavcnik (2002) shows that about one third of aggregate productivity growth of Chilean plants over the period 1979 to 1986 can be explained by this type of reallocation. Similarly, Bernard and Jensen (2004a) find that about 40% of total factor productivity growth can be attributed to a redistribution of resources across firms in the US manufacturing sector during the late 80s and early 90s.

Despite the fact that there is clear evidence for selection playing an important role in explaining economic growth, the growth literature based on selection is quite limited. The first papers incorporating selection as a growth mechanism were developed in the early 80s. Being based on evolutionary economics literature, most of these contributions are focused on bounded rationality. Gabler and Licandro (2008) and Luttmer (2007) are the first to provide models of endogenous growth through selection of successful firms and imitation by entrants based on rational expectations. When calibrated to US data, both papers find that a significant part of output growth can be attributed to selection and imitation, about one-fifth in the former and one-half in the latter. Even though the two papers are similar, they differ in the setup: Luttmer (2007) works in a framework of monopolistic competition, and emphasizes on matching the observed size distribution of firms, while the model of Gabler and Licandro (2008) is based on an environment of perfect competition.
This paper develops a model of trade with heterogeneous firms combined with endogenous growth. Endogenous growth is generated by idiosyncratic firm productivity improvements, selection of existing firms and imitation of surviving firms by entrants, as in Gabler and Licandro (2008) and Poschke (2007). Hence, in this model, both the mechanism through which the economy is affected by opening up to costly trade and the mechanism generating growth work through a channel of selection, i.e. high productivity firms expand their market share and low-productivity firms either lose market share or exit the market. Concerning the trade component, the model is based on Melitz (2003). The aim of the paper is to analyze how trade affects growth through the specific channel of selection. Moving from a closed economy to an economy with costly trade makes the growth rate in the economy permanently increase, because the effect of selection and imitation on growth is amplified by the selection process that is due to trade.

The following mechanism underlies the result. The existence of fixed costs of production makes it impossible for firms with low productivity to generate positive profits. This implies a cutoff productivity level below which exit is optimal. The idiosyncratic productivity shock hitting incumbent firms is more likely to push firms with already low productivity levels below the cutoff. This means that the average productivity of the whole economy, and also the distribution of incumbents, shift to the right. To ensure that there are always new firms replacing the exiting ones, entry takes place. In order to always have entrants above the cutoff productivity level, the distribution of entrants has to follow the distribution of incumbents in its movement to the right. This is achieved by allowing entrants to imitate imperfectly successful incumbents. Therefore growth is sustainable.

If the economy opens up to trade, and hence gives firms the opportunity to export their product, which is assumed to require a payment of a fixed cost, the cutoff productivity level increases. This comes from the fact that only the most productive firms will be able to afford paying this fixed fee for exporting, while less productive firms serve only the domestic market. This leads to two effects. First, since in an open economy, consumers have the choice to consume more varieties than in the closed economy, and since preferences are such that there is love for variety, the demand in one country for each variety goes down. Hence the low productivity firms face a decrease in market share, while high productivity firms increase their market share (because they serve the domestic and the foreign market). Second, the competition on the domestic factor market for the only factor of production, which is labor, increases. This is because the fixed costs for exporting are measured in labor, and hence those firms which export demand more of this limited factor. The increased labor demand leads to higher real wages, and forces the least productive firms to exit the market. In other words, the cutoff productivity level for production is higher in an economy where trade is possible but requires a fixed initial investment, than in an economy where no exchange is possible. For the growth mechanism, this means that there is more selection, and hence the average productivity increases at a faster rate than in a closed economy.

The remaining of the paper is organized as follows: in Section 2 the setup for the closed economy
is presented, and in Section 3 the model is extended to the open economy case with trade between two symmetric countries. Section 4 provides a calibration, numerical solution and results of the model. Section 6 concludes.

2 Closed Economy

2.1 Demand

There is a continuum of households in the economy. Each household lives forever and inelastically supplies labor. The population does not grow, and aggregate labor supply is normalized to one. Preferences of the representative household are given by a CES utility function:

\[ U = \sum_{t=0}^{\infty} \beta^t \ln(C_t), \]

where

\[ C_t = \left( \int_{\omega \in \Omega} q_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}. \]

Households consume different varieties \( \omega \), and the total set of varieties is given by \( \Omega \). Different varieties are substitutes, and the elasticity of substitution between any two varieties is given by \( \theta > 1 \). The discount factor is \( \beta \), with \( \beta \in (0, 1) \). Aggregate expenditure in the economy is given by \( E_t = C_t P_t \), where \( P_t \) is the aggregate price level:

\[ P_t = \left( \int_{\omega \in \Omega} p_t(\omega)^{1-\theta} d\omega \right)^{\frac{1}{\theta}}. \]

The static consumers problem is given by maximizing the consumption of each variety, taking into account the aggregate expenditure. Solving this maximization problem yields the households demand for each variety \( \omega \)

\[ q_t(\omega) = \left( \frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t. \]  

(1)

Hence, the optimal expenditure for variety \( \omega \) is

\[ e_t(\omega) = \left( \frac{p_t(\omega)}{P_t} \right)^{1-\theta} E_t. \]

Households also choose the optimal intertemporal allocation of consumption by maximizing the lifetime utility, taking into account their budget constraint. They can consume or invest in shares of a mutual fund, that pays a return \( r_t \), which is the real interest rate. Firms in the economy generate aggregate profits, and since firms are owned by households, profits are transfered as dividends,
allowing consumers to shift consumption over time. This eliminates any liquidity constraints of firms. Solving the dynamic optimization problem yields the standard Euler Equation, which defines the growth rate of consumption

\[ g_t^c = \frac{C_{t+1}}{C_t} = \beta(1 + r_t). \]  

(2)

This implies that the gross real interest rate in the economy is given by \( 1 + r_t = (1 + g_t^c)/\beta \).

### 2.2 Supply

There is a continuum of firms in the economy, each choosing to produce a different variety \( \omega \). Technology for a firm with productivity \( \varphi \) is given by

\[ q_{\omega,t}(\varphi) = \varphi_{\omega,t}(l_{\omega,t} - f^p). \]  

(3)

Marginal costs are constant and \( f^p \) is the fixed cost of production. Firms are heterogeneous in their productivity levels \( \varphi \). Every period each firm receives a shock to its productivity. This idiosyncratic shock follows a random walk

\[ \ln(\varphi_{\omega,t+1}) = \ln(\varphi_{\omega,t}) + \eta_{\omega,t+1}. \]  

(4)

The idiosyncratic productivity shock is assumed to be normally distributed, \( \eta_{\omega,t} \sim N(0, \sigma^2_\eta) \), i.e. the expected growth rate of firm specific productivities is zero for each firm. The subscript \( \omega \) is dropped from now on, because each firm produces a different variety, even if two firms have the same productivity. Firms which have the same \( \varphi \) charge the same price, hire the same amount of labor and hence make the same profits, even if they supply different varieties. Profits of a firm in period \( t \) are given by:

\[ \pi_t(\varphi) = q_t(\varphi)p_t(\varphi) - w_t l_t(\varphi). \]

It follows from the profit maximization problem that a firms with productivity \( \varphi \) will charge a price

\[ p_t(\varphi) = \frac{\theta}{\theta - 1} \frac{w_t}{\varphi_t}. \]  

(5)

Plugging the optimal price into the optimal expenditure from the household problem yields the firms revenue:

\[ e_t(\varphi) = E_t \left( \frac{\theta - 1}{\theta} \frac{\varphi_t}{w_t} P_t \right)^{\theta - 1}. \]  

(6)

It follows that profits can be rewritten as

\[ \pi_t(\varphi) = \frac{1}{\theta} e_t(\varphi) - f^p. \]  

(7)

From now on nominal wages are normalized to one, i.e. \( w_t = 1 \) for all periods.
2.3 Firm Entry and Exit

The firm dynamics are based on the model of Hopenhayn (1992). Every existing firm receives an idiosyncratic shock in each period as is specified in equation (4). This means that some firms will decide to exit the market because their productivity is lower than a certain threshold $\phi^*$, below which producing would yield a negative firm value. The probability density function of incumbent firms is given by $\mu_t(\phi)$. No specific distributional form is assumed since it is determined endogenously in equilibrium. The mean and the variance are denoted by $x_i^t$ and $\sigma_i^2$ respectively.

Entering firms have to pay a sunk entry cost $f_e$, and are less productive on average than incumbent firms even though they try to imitate successful incumbents. They start with a productivity level which they draw from a log-normal distribution $\gamma_t(\phi)$ with a mean $x_e^t$ and variance $\sigma_e^2$. The imitation process is modeled as in Poschke (2007). The mean of the entrants productivity distribution follows the productivity of the best incumbent, $\phi_{t}^{\text{max}}$, with a constant distance $\kappa > 0$:

$$x_e^t = \phi_{t}^{\text{max}} - \kappa, \quad (8)$$

where $\phi_{t}^{\text{max}}$ is defined as being the average of the best 5 percent of all producing firms.

The timing of the economy is defined as follows: A firm takes the decision to exit at the beginning of period $t$. The relevant threshold for the decision to produce in $t$ is given by $\phi^*_t$. If the decision has been taken to produce in a given period, then an incumbent gets a new productivity draw, pays the fixed costs of production $f_p$ and produces. The entry decision of new firms is also taken at the beginning of period $t$. If entry occurs, then the entrant has to pay the fixed entry costs $f_e$, gets its initial productivity draw out of the distribution $\gamma_t(\phi)$, pays the fixed costs of production $f_p$ and produces. Both fixed costs $f_p$ and $f_e$, are payed in labor units.\(^2\) See Appendix A for a graphical illustration of the timing assumptions in the economy.

The value function of a firm with productivity level $\phi$ is given by

$$V(\phi) = \max_{\phi} \left\{ \pi(\phi) + \frac{1}{1+r} \max \left\{ \int_{0}^{\infty} V(\phi') \nu^{\eta}(\phi'/\phi) d\phi', 0 \right\} \right\}, \quad (9)$$

where $\nu^{\eta}$ is the probability density function of the exponential of the idiosyncratic productivity shock $e^{\eta_{t-1}}$. This means that $\nu^{\eta}(\phi'/\phi)$ is the probability that a firm with productivity $\phi$ today receives a shock such that it has a productivity $\phi'$ tomorrow.

**Free exit:** Some firms decide to exit the market because their productivity does not ensure them a positive expected future value. The cutoff productivity level is determined by $\phi^*_t = $\(^2\)Fixed costs not constant, but evolve over time. Since they are given in terms of labor, and nominal wages are normalized to one, this means that they are increasing in terms of consumption.
\[ \inf \{ \varphi_t : V(\varphi_{t+1}) \geq 0 \} \]. The free exit condition is thus given by

\[ V(\varphi^*) = \max_p \left\{ \pi(\varphi^*) + \frac{1}{1 + r} \max \left\{ \int_0^\infty V(\varphi') \nu^*(\varphi' / \varphi^*) d\varphi', 0 \right\} \right\} = 0 \quad (10) \]

**Free entry**: A fixed sunk cost \( f_e \) has to be payed by each firm which wants to start production in the market. New firms will enter the market until the net value of entering is driven to zero. It follows that the free entry condition is given by

\[ V_e = \int_0^\infty V(\varphi) \gamma(\varphi) d\varphi = f_e. \quad (11) \]

**Transition function**: In every period there are incumbent firms with distribution \( \mu_t(\varphi) \) and entrants with distribution \( \gamma_t(\varphi) \). In the following period the 'new' PDF of incumbents will be the one of the old surviving incumbents (i.e. those firms that have a productivity level higher than the cutoff level), plus the new entrants. Hence the transition function for the distribution of incumbent firms is given by

\[ N' \mu'(\varphi') = N \int_{\varphi^*}^\infty \nu^*(\varphi' / \varphi) \mu(\varphi) d\varphi + N' e \gamma(\varphi'), \quad (12) \]

where \( N \) is the number of incumbents, and \( N_e \) the number of firms entering the market.

### 2.4 Aggregation

The aggregate productivity level is denoted as \( \bar{\varphi} \). It is also the average productivity weighted by relative output shares and is given by: \(^3\)

\[ \bar{\varphi}_t = \left( \int_0^\infty \varphi^{\theta - 1} \mu_t(\varphi) d\varphi \right)^{\frac{1}{\theta - 1}}. \quad (13) \]

Using the definition of the aggregate price level given above, the optimal price chosen by firms and the definition of the cutoff level, the aggregate price level can be expressed as

\[ P_t = \left( \int_0^\infty p_t(\varphi)^{1 - \theta} N_t \mu_t(\varphi) d\varphi \right)^{\frac{1}{1 - \theta}} = \frac{\theta}{\theta - 1} N_t^{\frac{1}{\theta - 1}} \bar{\varphi}^{-1}. \quad (14) \]

Aggregate output is

\[ Q_t = \left( \int_0^\infty q_t(\varphi)^{\theta - 1} N_t \mu_t(\varphi) d\varphi \right)^{\frac{1}{\theta - 1}} = q(\bar{\varphi}_t) N_t^{\frac{1}{\theta - 1}}, \quad (15) \]

\(^3\)The weight is \( \frac{q(\varphi_t)}{q(\bar{\varphi}_t)} \), which is the firms relative output shares.
and aggregate profits are
\[ \Pi_t = \int_0^\infty \pi_t(\varphi) N_t \mu_t(\varphi) d\varphi = N_t \pi_t(\tilde{\varphi}_t). \]

### 2.5 Equilibrium

The following points have to be fulfilled in any equilibrium of the economy:

- **Consumers** choose optimally the quantity consumed for each variety and the number of assets they hold, such that the Euler equation is fulfilled. Total asset holdings must equal total firm value.
- **Firms** choose optimally the price for each good given the demand they face from households.
- **Exit** is optimal and given by the free exit condition (10).
- **Entry** is optimal and given by the free entry condition (11). Firms enter as long as they can cover the entry costs \( f_e \).
- **Labor market clearing** implies that the supply of labor by households has to be equal to the labor demand by incumbents and entrants. In this economy fixed costs \( (f_p \text{ and } f_e) \) are also paid by labor. Thus total labor in the economy must be equal to the amount of labor used in production \( L^p_t \) and in paying the fixed entry costs \( L^e_t \), i.e. \( L = L^p_t + L^e_t \). The amount of labor used to pay the fixed costs of production can be determined by
\[ L^p_t = E_t - \Pi_t, \]
and the amount of labor used to pay the fixed costs of entry is
\[ L^e_t = N^e_t f_e. \]

**Firms distribution** \( \mu_t(\varphi) \) is determined endogenously by the transition function (12).

**Aggregate stability condition** implies that the number of firms in the economy \( N_t \) is stable in the equilibrium, i.e. the number of exiting and entering firms has to be equal:
\[ N_t \int_{\tilde{\varphi}_t}^{\varphi^*} \mu_t(\varphi) d\varphi = N^e_t. \]

This determines endogenously the number of firms producing in the economy in each period.

### 2.6 Balanced Growth Path

The balanced growth path (BGP) is defined as a state of the economy in which aggregate productivity, consumption and output grow at a constant rate \( g \), aggregate prices decrease at the same
constant rate, the distribution of firm productivities shifts up at steps of $g$, its shape is invariant\textsuperscript{4}, and aggregate expenditures, aggregate profits, the number of firms, the number of entrants and the interest rate are constant. The economy can then be stationarized, and to distinguish it from the growing economy, stationarized variables are denoted with a hat. The relevant equations for the BGP, i.e. the equations that have to be rewritten in stable terms, are the law of motion of productivity (4), the value function (9), the transitions function for the distribution of productivities (12), the free exit condition (10), and the free entry condition (11).

The random walk of productivities (4) gets a downward drift in the stationarized economy. The distribution shifts to the right every period by a step of size $g$, but the idiosyncratic productivity shock is such that it has a zero mean, i.e. a firm does not expect its productivity to change. Hence in expectations each firm has a decreasing productivity relative to the overall distribution:

$$\ln(\hat{\varphi}_{\omega,t+1}) = \ln(\hat{\varphi}_{\omega,t}) - g + \eta_{\omega,t+1}. \quad (20)$$

Combining equation (7) and (6), firm specific profits can be rewritten as

$$\pi(\hat{\varphi}) = \frac{1}{\theta} \left( \frac{\theta - 1}{\theta} \right)^{\theta - 1} k^{\theta - 1} - f^p, \quad (21)$$

where $k = EP^{\theta - 1}$. Substituting this expression into the value function (9), and using the Euler Equation (2) yields a stationary expression for the value function of a firm

$$v(\hat{\varphi}) = \max_p \left\{ \pi(\hat{\varphi}) + \frac{\beta}{g} \max \left\{ \int_0^\infty v(\hat{\varphi}'\nu^\eta(\hat{\varphi}' / \hat{\varphi})d\hat{\varphi}', 0 \right\} \right\}. \quad (22)$$

Applying the same method, the free exit condition (10) in the balanced growth path is

$$v(\hat{\varphi}^*) = \max_p \left\{ \pi(\hat{\varphi}^*) + \frac{\beta}{g} \max \left\{ \int_0^\infty v(\hat{\varphi}'\nu^\eta(\hat{\varphi}' / \hat{\varphi}^*)d\hat{\varphi}', 0 \right\} \right\} = 0, \quad (23)$$

the free entry condition (11) is

$$v^e = \int_0^\infty v(\hat{\varphi})\gamma(\hat{\varphi})d\hat{\varphi} = f^e, \quad (24)$$

and the transition function (12) can be rewritten as

$$\mu(\hat{\varphi}') = \int_{\hat{\varphi}^*}^\infty \nu^\eta(\hat{\varphi}' / \hat{\varphi})\mu(\hat{\varphi})d\hat{\varphi} + \frac{N^e}{N}\gamma(\hat{\varphi}') \quad (25)$$

Given these equations (20)-(25), the balanced growth path of the closed economy can be solved

\textsuperscript{4}Even though the evolution firm-specific productivities follows a random walk, the distribution of firms is stationary. Its variance remains finite over time since exit takes place mostly in the lower part of the distribution and since the probability of surviving decreases with the age of the firm. For more details see Poschke (2007).
2.7 Welfare

In this economy, welfare has a static and a dynamic component. On the one hand consumers have a positive utility from consuming. On the other hand they also benefit from consumption growth in future periods. Overall welfare is given by:

\[ W = \frac{C}{1 - \beta(1 + g)}. \] (26)

The static component of overall welfare is given by \( W^s = \frac{C}{1 - \beta} \), and is simply the discounted present value of consumption, not taking into account that the economy is growing. Dynamic welfare is then the difference between overall welfare and static welfare, i.e. \( W^d = W - W^s \).

3 Open Economy

In order to analyze the impact of trade on economic growth, the previous setup is adjusted to an open economy framework. Only trade between two symmetric countries is considered for simplicity. An extension to a larger number of countries trading with each other does not alter the main results.\(^5\)

The assumption of symmetry implies that both countries have the same wage, which is normalized to one, and that the aggregate variables of both countries are the same. Another assumption that is made, is that exporting firms face an additional fixed cost \( f_x \) for exporting in every period they serve the foreign market, and also variable, iceberg type, trade costs \( \tau \). The existence of fixed costs to exporting is crucial to the results of the model, because otherwise the only effect of trade on the economy is an increase of consumers welfare due to a rise in the number of varieties available for consumption as in Krugman (1980). There exist several empirical studies which find that firms face fixed costs to enter the export market, for example Bernard and Jensen (2004b) for the US.

On the demand side there are no changes in the setup of the economy due to opening up the economy. Consumers still face the same maximization problem subject to the same constraints, which means that the demand for each variety is determined as in the closed economy and is given by equation (1). On the other hand firms now also have to make an additional decision: after receiving their productivity draw firms have to evaluate whether they want to pay the fixed investment to export, or only serve the domestic market.

\(^5\)See Melitz (2003) for trade between \( n \) number of symmetric countries
3.1 Supply

The production function is the same as before, and firms that sell only in the domestic market pay the fixed costs $f^p$, but firms which also enter the export market now pay additionally the fixed cost $f^x$. The profit function changes because now profits can be generated from local and from foreign sales. Production, prices, the amount of labor used and profits for the local market are denoted by $\pi^d$, $q^d$, $p^d$, $l^d$, and for the exporting market by $\pi^x$, $q^x$, $p^x$, $l^x$.

$$\pi^d_t(\varphi) = q^d_t(\varphi)p^d_t(\varphi) - l^d_t(\varphi)$$

$$\pi^x_t(\varphi) = q^x_t(\varphi)p^x_t(\varphi) - l^x_t(\varphi)$$

The total amount of labor spent in production by a firm with productivity $\varphi$ is $l_t(\varphi) = l^d_t(\varphi) + l^x_t(\varphi)$, where $l^x_t = 0$ if the firm sells only in the domestic market.

The price for domestic sales $p^d_t$ is the same as in closed economy, i.e. given by equation (5), but a firm that exports will set higher prices in the export market because of the per unit trade costs:

$$p^x_t(\varphi) = \tau \left( \frac{\theta}{\theta - 1} \right) \frac{w_t}{\varphi_t}. \quad (27)$$

Overall profits of a firm with productivity $\varphi$ in period $t$ are given by

$$\pi_t(\varphi) = \pi^d_t(\varphi) + \max \{0, \pi^x_t(\varphi)\}. \quad (28)$$

where $\pi^d_t(\varphi_t)$ is given by equation (5), and $\pi^x_t(\varphi) = \frac{1}{\theta} c^x_t(\varphi) - f^x$, with $c^x_t(\varphi) = \tau^{1-\theta} c^d_t(\varphi)$.

3.2 Firm Entry and Exit

The value function of a firm with productivity $\varphi$ is given by equation (9). Notice that profits that enter the value function are not the same as in closed economy, because they now consist of domestic and export sales. In the open economy there are two cutoff levels, one for producing $\varphi^*_t$ and one for exporting $\varphi^{x*}_t$. As in the closed economy, the production cutoff level is defined as $\varphi^*_t = \inf \{ \varphi_t : V_t(\varphi_{t+1}) \geq 0 \}$, and the productivity cutoff level for entering the export market is $\varphi^{x*}_t = \inf \{ \varphi_t : \varphi_t > \varphi^*_t \}$. As in the closed economy, the free entry condition is again given by equation (11), and the transition function of the distribution of incumbents by equation (12).

The timing in the open economy is the same that in the closed economy, except of the decision to enter the export market. Once the firms, incumbents and entrants, got their productivity draw for a given period, they decide whether to export or not in this period. Entering the export market takes place if $\varphi_t \geq \varphi^{x*}_t$. 


3.3 Aggregation

Aggregate productivity is as before given by the weighted average productivity, with the weight being relative output shares. It can not be defined in the same way as in the closed economy, because equation (13) does not take into account the higher market share of exporting firms. In order to do so, it has to be considered that some firms export, and some firms serve only the domestic market. Hence there are two aggregate productivity levels, $\tilde{\phi}_d^t$ for all firms (but taking into account only domestic market shares), and $\tilde{\phi}_x^t$ for exporting firms only (including only exporting market shares):

$$\tilde{\phi}_d^t = \left( \int_0^\infty \phi_t^{\theta-1} \mu_t(\phi_t) d\phi_t \right)^{\frac{1}{\theta-1}},$$

$$\tilde{\phi}_x^t = \left( \frac{1}{1 - M(\phi_t^{\tau_\phi})} \int_0^\infty \phi_t^{\theta-1} \mu_t(\phi_t) d\phi_t \right)^{\frac{1}{\theta-1}},$$

where $1 - M(\phi_t^{\tau_\phi})$ is the ex-ante probability for each firm to draw a productivity level higher than the exporting cutoff. The total aggregate productivity level, which also reflects the relative market shares, is then given by:

$$\tilde{\phi}_t = \left( \frac{N_t}{N_t + N_t^x} \right)^{\theta-1} \tilde{\phi}_d^t + \left( \frac{1}{\tau} \tilde{\phi}_x^t \right)^{\theta-1},$$

(29)

where $N_t^x$ is the number of firms exporting, or the number of varieties exported to the other country.\(^6\) The variable trade costs $\tau$ reflect the output shrinkage linked to exporting. Since every exporting firm is also producing for the domestic market, the total number of firms producing in the economy is $N_t$. Since additionally to domestic varieties, the consumers also have access to imported varieties, the total mass of different varieties available to a consumer is $N_t + N_t^x$.

The aggregate prices are now given by

$$P_t = \frac{\theta}{\theta-1} (N_t + N_t^x)^{\frac{1}{\theta-1}},$$

and aggregate profits by

$$\Pi_t = \frac{1}{\theta} E_t - f^p N_t - f^x N_t^x.$$

3.4 Equilibrium

Consumers choices, exit and entry decisions, the firms distribution and the aggregate stability condition are determined in the same way as in the closed economy equilibrium. In addition to this, the following points have to be fulfilled as well.

Export decisions are taken optimally by firms, which only export if they make non-negative profits

\(^6\)Note that by the assumption of symmetry, this is also equal to the number of varieties imported to the domestic country.
form serving the foreign market, i.e. if $\pi_x > 0$.

**Firms** which export additionally choose optimally the price they charge for the goods they export facing foreign demand.

**Labor market clearing** implies that the supply of labor by households has to be equal to labor demand by incumbents and entrants. In this economy fixed costs ($f_p$, $f_e$ and $f_x$) are also paid by labor. Thus total labor in the economy must be equal to the amount of labor used in production $L^p_t$ and the fixed entry costs $L^e_t$, i.e. $L = L^p_t + L^e_t$, where the labor used in production includes the domestic and exporting part. The amount of labor used to pay the fixed costs of production can be determined by equation (17), the amount of labor used to pay the fixed costs of entry by equation (18). Labor used to pay fixed costs of exporting can be determined by

$$L^x_t = N^x_t f^x.$$  \hspace{1cm} (30)

### 3.5 Balanced Growth Path

The balanced growth path is defined in the same way as in the closed economy, except of the expression for overall profits, now given by (28). This is the only change to these equations, and hence, as in the closed economy, the value function is given by equation (22), the free entry condition by equation (24), and the free exit condition by equation (23). The transition function of the incumbents distribution is given by equation (25). As before, given these equations, the balanced growth path can be solved numerically, and welfare is determined by equation (26).

### 4 Solution

#### 4.1 Calibration

In this section, parameter values of the open economy model are calibrated to the U.S. manufacturing sector in order to derive the quantitative conclusions of the selection and imitation mechanism on the growth rate of productivity. The parameters that need to be calibrated are the discount factor $\beta$, the elasticity of substitution $\theta$, the fixed costs of production $f_p$, entry $f_e$ and exporting $f_x$, the variable exporting costs $\tau$, the variance of the productivity distribution of entrants $\sigma^2_e$, the variance of the idiosyncratic productivity shock hitting incumbents $\sigma^2\nu$, and the imitation parameter $\kappa$. Common values from the literature are assigned to $\beta$ and $\theta$. All other parameters are jointly chosen by minimizing the distance between some facts observable in the data and the equivalent moment of the model by using a genetic algorithm as described by Dorsey and Mayer (1995).

The facts observed in the data and used for the calibration are the following: the seven-year survival rate of entrants, the exit rate, the average firm size, the size of entrants relative to incumbents, the annual growth rate of the economy, the proportion of exporters, and the size advantage
of exporters. The last two facts help to determine the trade costs $f_x$ and $\tau$. The size of entrants relative to incumbents allows to find the imitation parameter $\kappa$, since it establishes a relationship between the distribution of incumbents and entrants. The seven-year survival rate of entrants, the exit rate and the average firm size give some good indications about the firm dynamics and scale, and thus help to calibrate the parameters $f^e$, $f^p$ and $\sigma^2_e$. Finally, the growth rate of output determines the variance of the idiosyncratic productivity shock $\sigma^2_\eta$.

The survival rate of firms seven years after entry in the market is 48 percent according to Bartelsman et al. (2004) for the U.S. manufacturing sector. They also find that the exit rate averaged over the time period 1989 to 1997 is approximately 8 percent, and that the size in terms of employment of new firms is only 18 percent of incumbents size. Using the same dataset, Bartelsman et al. (2003) show that the average size in terms of employment of manufacturing firms in the U.S. is 80.3. Concerning the facts related to trade, Bernard et al. (2003) report that the proportion of exporters is 21 percent for the U.S. analyzing the 1992 Census of Manufacturers, and show that exporting firms have a size advantage of 4.8 for the ratio of average U.S. sales. This measure is the ratio of average output of exporting plants to the average for non-exporting plants, taking only domestic sales into account for both exporters and non-exporters. The annual growth rate is set to 3 percent, which is the average output growth rate in the 1990s according to the NIPA tables. The calibration targets and the values generated by the model are given in Table 1.

All targets are reasonably well matched by the model statistics. The slightly too high proportion of exporters in the model economy can easily be explained by a miss reporting of mainly small firms about their exporting activity, as argued in Bernard et al. (2003). \footnote{This would imply that the model should also underestimate the productivity advantage of exporters. This is however not the case. The model yields a too high productivity premium for exporters, and exporters also produce a too large share of output. An explanation for this is that the steady state analysis implicitly assumes that total imports equal total exports. In the U.S. there has been a large trade deficit over the last decades, thus the observed data concerning trade are very likely not be very close to steady state values. This might result in the difficulties of the model to match exactly exporter facts concerning their size and their overall importance in the economy.}

The parameter values resulting from the calibration are summarized in Table 2. The discount factor $\beta$ is set to 0.95, which implies an annual interest rate of approximately 5 percent. For $\theta$,

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Target (U.S.)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>7-year survival rate of entrants</td>
<td>48%</td>
<td>44.22%</td>
</tr>
<tr>
<td>Exit rate</td>
<td>8%</td>
<td>6.7%</td>
</tr>
<tr>
<td>Average firm size (employment)</td>
<td>80.3</td>
<td>82.81</td>
</tr>
<tr>
<td>Size of entrants relative to incumbents</td>
<td>18%</td>
<td>17.58%</td>
</tr>
<tr>
<td>Growth rate</td>
<td>3%</td>
<td>2.99%</td>
</tr>
<tr>
<td>Proportion of exporters</td>
<td>21%</td>
<td>23.85%</td>
</tr>
<tr>
<td>Size advantage of exporters (Ratio domestic sales)</td>
<td>4.8</td>
<td>4.70</td>
</tr>
</tbody>
</table>
the elasticity of substitution between any two varieties, the value adopted from the literature is 3.8. It is taken from Bernard et al. (2003), who obtain this value by calibrating their model to fit U.S. plant and macroeconomic trade data. All fixed costs are given in units of labor, i.e., give the number of workers necessary in order to pay the specific fixed costs. In the open economy specification these values imply that employment in fixed costs of production, entry and exporting represent 1.83%, 4.24% and 1.70% of total employment respectively. The variable trade costs \( \tau \) take the value 1.14. This means that in order to sell one unit abroad an exporting firm has to produce 1.14 units. Concerning the imitation parameter, it represents the productivity difference between the best 5% of all producing firms and the average entrant.

### Table 2: Parameter values

<table>
<thead>
<tr>
<th>Parameters from Literature</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of substitution ( \theta )</td>
<td>3.8</td>
</tr>
<tr>
<td>Discount factor ( \beta )</td>
<td>0.95</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters from Calibration</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed costs of production ( f^p )</td>
<td>1.66</td>
</tr>
<tr>
<td>Fixed costs of entry ( f^e )</td>
<td>57.5</td>
</tr>
<tr>
<td>Fixed costs of exporting ( f^x )</td>
<td>6.5</td>
</tr>
<tr>
<td>Variance of entrants distribution ( \sigma^2_e )</td>
<td>0.55</td>
</tr>
<tr>
<td>Variance of incumbents shock ( \sigma^2_\eta )</td>
<td>0.15</td>
</tr>
<tr>
<td>Variable costs of exporting ( \tau )</td>
<td>1.14</td>
</tr>
<tr>
<td>Imitation parameter ( \kappa )</td>
<td>3.4</td>
</tr>
</tbody>
</table>

### 4.2 Closed and Open Economy Comparison

In this section the solution of the model is discussed. Appendix B describes the algorithm used to obtain this solution. The aim of the paper is to analyze how trade affects growth. Thus the issue is to compare the results of the closed economy model with those of the open economy.

### Table 3: Model results

<table>
<thead>
<tr>
<th>Closed vs. Open Economy</th>
<th>Closed Economy</th>
<th>Open Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth Rate</td>
<td>2.84%</td>
<td>2.99%</td>
</tr>
<tr>
<td>Weighted Average Productivity</td>
<td>2.59</td>
<td>2.83</td>
</tr>
<tr>
<td>Cutoff Level for Production</td>
<td>0.68</td>
<td>0.74</td>
</tr>
<tr>
<td>Cutoff Level for Exporting</td>
<td>-</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Table 3 summarizes the most important results concerning productivity. Unambiguously, opening up to trade yields a higher growth rate, cutoff level for production and weighted average pro-

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8This value is lower than usually in the literature, hence the resulting markup is higher. However, the presence of fixed costs in the model justifies this choice of \( \theta \). See Ghironi and Melitz (2005) for a more detailed discussion.
ductivity. The increase in the growth rate is of 15 basis points. Considering that the effect on the growth rate is due only to an increase in selection, disregarding any other source of variation, this is a substantial change. Table 4 gives the percentage change resulting from opening up the economy from autarky for some important aggregate variables. The 15 basis point increase of the growth rate implies that the growth rate is more than 5 percent higher in the open economy. The average firm size also rises and the number of firms in the economy decreases, i.e. there are less firms in the economy once trade is introduced, but those firms are more productive and bigger. Less labor is used for paying fixed production and entry costs, which is due to the decrease in number of firms and entrants. The decrease in number of entering firms is smaller than the reduction in total number of firms, hence the exit rate increases. This higher turnover of firms comes from more entry due to higher expected value, and more exit due to tougher competition for the factor of production labor, which is also used to pay fixed exporting costs in the open economy. As a consequence of the previous results, the aggregate price level is lower and the aggregate output higher in the economy that allows for costly trade. A higher aggregate output level combined with a higher growth rate induces an increase in overall welfare: it rises by 15 percent.

Table 4: Opening up to trade

<table>
<thead>
<tr>
<th>Shift from Closed to Open Economy</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth Rate</td>
<td>+5.3%</td>
</tr>
<tr>
<td>Weighted Average Productivity</td>
<td>+9.3%</td>
</tr>
<tr>
<td>Average firm size</td>
<td>+17.9%</td>
</tr>
<tr>
<td>Exit Rate</td>
<td>+21.8%</td>
</tr>
<tr>
<td>Fraction of employment in fixed production costs</td>
<td>-18.7%</td>
</tr>
<tr>
<td>Fraction of employment in fixed entry costs</td>
<td>-1.8%</td>
</tr>
<tr>
<td>Number of firms</td>
<td>-21.4%</td>
</tr>
<tr>
<td>Number of entrants</td>
<td>-1.8%</td>
</tr>
<tr>
<td>Aggregate prices</td>
<td>-8.6%</td>
</tr>
<tr>
<td>Aggregate output</td>
<td>+7.7%</td>
</tr>
<tr>
<td>Welfare</td>
<td>+15.0%</td>
</tr>
</tbody>
</table>

Figure 1 illustrates the stationary distributions of all producing firms and entrants in the open economy case. The entrants distribution is lagging behind, while the distribution of all producing firms is more to the right. The imitation parameter $\kappa$ is the distance between the best producing firms and the average of entrants.

The difference in the productivity distribution of the closed and the open economy is shown in Figure 2. The latter is shifted to the right, which is due to the fact that some low productivity firms are forced to exit the market when the economy opens up for trade. This is because only the most productive firms can afford to pay the fixed costs of exporting. Firms with a low productivity serve only the domestic market. They loose market share because of foreign firms selling their products.
in the domestic market, and the increased competition on the factor market for labor pushes the real wages up.

Figure 1: Stationary Distribution Open Economy - Entrants and all Firms

Figure 2: Stationary Distribution of all Firms - Open and Closed Economy
4.3 Trade Liberalization

So far the analysis consisted of comparing two different steady states, one of autarky and one of costly trade. More realistically, this section focuses on trade liberalization, i.e. the decrease in trade costs. How does a cheaper access to export markets affect the growth rate of the economy? As can be seen in figures 3 and 4, decrease in either of the trade costs in the model leads to higher growth.\(^9\)

![Figure 3: Growth Effect of a Change in Fixed Costs of Exporting](image)

The intuition behind this effect is simple. When variable or fixed costs of trade decrease, then the productivity level necessary to derive positive profits from export markets is lower. Hence, the export cutoff level, \(\varphi^x\), decreases and more firms have access to foreign markets. The production cutoff level, \(\varphi^p\), goes however in the opposite direction: the higher exposure to trade has as consequence that the firms with the lowest productivity levels in the economy have to exit the market. This has two implications. First, this directly increases the aggregate productivity level in the economy. Second, the higher cutoff level means that each firm has now a higher probability to be hit by a shock bad enough to be forced out of the market. This tougher selection makes the economy profit from an increase in its growth rate.

Concerning welfare, an increased liberalization of trade has a positive impact. Figure 5 shows

\(^9\)Note that the steps in the two figures come from the discretization necessary to compute the numerical solution. The results here are computed for 1000 grid points, and a substantially larger number would be needed in order to obtain a smooth line.
how welfare evolves with changes in fixed exporting costs.\textsuperscript{10} Static and dynamic welfare are disentangled in order to analyze the behavior of both separately. It becomes clear that both static and dynamic welfare increase with trade liberalization, which is simply a consequence from the increase of aggregate output and the growth rate due to trade opening. However, it also shows that the dynamic component of welfare is more important than the measure which does not take into account growth. Recall that the static welfare component is defined as being discounted lifetime consumption ignoring growth, while dynamic welfare is difference of between this same measure accounting for and ignoring growth. Thus the latter incorporates only the positive effect of being in a growing economy versus being in a zero growth environment. The figure shows very clearly that the more important welfare gain coming from trade liberalization is the one due to the increase in the growth rate of the economy. This observation shows that a discussion of the benefits of trade liberalization in models without growth is not suitable.

4.4 Competition and the Growth Effect of Trade

In the previous discussion it has become clear that trade-induced selection is an important factor which influences the growth rate of the economy. How this is related to competition aspects is explained in more detail in this section.

\textsuperscript{10}For this analysis, variable trade costs are assumed to be zero, i.e. $\tau = 1$. 

Figure 4: Growth Effect of a Change in Variable Costs of Exporting
There are two channels in this model economy through which the aggregate productivity increases. First, the factor market becomes more competitive. When international trade is introduced in the economy, those firms for which it is profitable to export want to increase the scale of their production in order to serve the foreign market. This, combined with the fact that fixed costs to enter the export market are paid in labor, increases the demand for the only factor of production. Real wages are pushed up by this new demand. For some firms, more explicitly for those with a low productivity, profits associated with this new real wage are not high enough to survive. The second channel consists of an increase in competition coming from the fact that there are new, more productive, competitors present in the market when the economy opens up to trade. As mentioned above, the number of domestic firms is lower in the open economy, and, depending on parameter values, the entry of foreign firms in the domestic market might lead either to more or to less firms selling their varieties in the domestic market. In the specific baseline calibration of the model, this overall effect is negative: less varieties are available for consumers in an economy that allows for trade. However, the new foreign competitors firms face, are more productive, since also in the foreign country only the most productive firms have access to the export market. This as such makes the weighted average productivity increase, and thus aggregate prices decrease. The decrease in aggregate prices directly implies that the relative price for each firm decreases, and hence the domestic demand for each variety goes down, i.e. all firms face a reduction in their market share. Firms with a low productivity might not make sufficiently high profits when facing the new demand for their good. The combined market exit of both channels leads then to an increase in aggregate
productivity.

However, the quite restrictive assumption of C.E.S. preferences has some important implications for the competition effects in this model. Since the elasticity of substitution is constant, it does not adjust to a change in the number of competing firms or prices. The markups charged by firms are constant, hence their prices do not vary with the increase in competition. Thus, there is no competition effect in the sense of price adjustments by firms.

The elasticity of substitution directly affects the growth rate. It is an indicator of how competitive the economy is. When $\theta$ is low, different varieties are only very imperfect substitutes, which implies a low degree of competitiveness in the market. Opposite to this, high values of $\theta$ stand for higher competitiveness. Thus different levels of elasticities imply different growth rates in the economy, because selection plays a more or less important role. Since the markup depends negatively on the demand elasticity, a low $\theta$ yields a high markup. In this case, since firms can charge a high markup, less productive firms can make profits which are high enough to stay in the market. This means that for low elasticities, selection does not play a big role in the economy. Hence, the higher the elasticity of substitution, the more competitive the market and the higher the growth rate.

![Figure 6: Effect of the Elasticity of Substitution on the Growth Differential](image)

Figure 6 shows how the growth rate difference between the open and closed economy varies with $\theta$. That this difference in growth rates is not constant but hump shaped comes from the fact that the trade-induced additional selection impacts the economy differently for different levels of competitiveness. The explanation for the increase in the range of relatively low values of $\theta$ is as
follows. Allowing for trade, as described above, leads to a decrease in the aggregate price level. Hence the relative price each producer charges goes up. With a low elasticity of substitution, consumers do not react a lot to this change in relative prices. Thus some less efficient firms can continue to survive in the market. When $\theta$ increases, the additional selection effect coming from trade plays an increasingly important role in the economy. The intuition behind the decrease in the growth differential in the range of relatively high values of the elasticity of substitution is similar: larger values of $\theta$ stands for more competition, i.e. for an economy where the selection mechanism is important. In this case, the selection induced by trade has no, or a too small effect on the economy. Thus the growth differential decreases with an increased substitutability between goods.

5 Conclusion

This paper has analyzed the impact of opening up an economy form autarky to costly trade on the productivity growth rate of the economy. For this purpose an endogenous growth model with firm heterogeneity and intra-industry trade has been developed. Both the channel for growth and for trade work through a mechanism of selection of more productive firms into the domestic and export market, while the least productive firms are forced to exit. In the model, incumbent firms are hit every period by an idiosyncratic productivity shock and entrants are able to partly imitate successful incumbents. This generates sustained productivity growth. Introducing the possibility for firms to trade their products by paying a fixed cost has the effect to increase the minimum productivity level required for production. This makes selection tougher, i.e. forces more low-productivity firms to give up their position in the market, and hence increases the growth rate of the economy.

For the last years there has been an ongoing debate about the benefits and shortcomings of globalization. One of the main fears is that opening up to trade could force some firms to close down. The model developed in this paper does not allow for a general statement about the relation between trade and growth. However one very important conclusion can be drawn: considering the channel of the selection effect of trade on growth, countries that open up to trade will face closure of firms, but will gain in aggregate productivity and grow at a faster rate. It follows that in the short run, a protectionist policy could preserve some job opportunities. The long run consequences are however likely to be lower average productivity levels, higher prices and lower growth rates.
Appendix A. Timing

Appendix B. Algorithm

The algorithm used to obtain the numerical solution of the balanced growth path constructed in the following way.

First the state space of productivities is discretized, which means that a grid of productivities $\hat{\psi}$ is created. The number of grid points is set to 200. A higher number of grid points does not have an implication on the main results of the model. Then the variable $k$ and the growth rate $g$ are guessed. For a given $k$ and $g$, the transition probability matrix $\nu(\hat{\psi}/\hat{\psi})$, denoted $T$, can be computed, taking into account the downward drift according to equation (20). The next step is to create the distribution of entrants $\gamma(\hat{\psi})$, which is assumed to be lognormal. Then the variable $k$ can be determined using the free entry condition, i.e. the $k$ is computed for which the free entry condition (24) holds, given $g$. This allows then to compute the value function (22) by value function iteration. Firms which get a negative value from production choose to exit, hence the cutoff productivity level $\hat{\psi}^*$ is known. The cutoff level allows then to create a transition probability matrix $T_x$ which includes exit. Using this, the stationary firm distribution, for given $g$, can be
obtained directly by $\mu = (I - T_x)^{-1}\gamma$. In the case of the open economy the decision of entering the export market has to be included. This is done by evaluating the profits for exporting, $\pi^x$, for every existing productivity level. Firms with $\pi^x > 0$ decide to export, and other firms only serve the domestic market. This also delivers the export market cutoff $\hat{\phi}^{*x}$. The profits made from exporting enter the overall profits which are used to compute the value function. The last step is to obtain the growth rate $g$. This is done via the imitation mechanism. The mean of the entrants distribution is normalized to zero, and the equilibrium growth rate is the one fulfilling equation (8).
References


