US Current Account Deficit, Extensive Margin and Firms Heterogeneity*

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1 Introduction

The rising interest of scholars on the deficit of the US Current Account is due to two main reasons. Firstly, evidence over the last decade shows a trend towards a strong worsening of the US Current Account position with the deficit accounting for almost 7% of total GDP of the country. On the other hand, if we expect an adjustment of the Current Account, we should observe a great depreciation of the US dollar with respect to the main currencies of the rest of the world like euro and yen, but time series on the US dollar do not show such a process, so generating something of a puzzle in International Macroeconomics.

Is this lack of great depreciation by the US dollar really a puzzle? Why do some emerging countries finance the US current account deficit through the acquisition of US treasuries and other liabilities? Is the current account deficit still sustainable for the economy of US, or is the adjustment of the current account inevitable? What could be the impact of a disruptive adjustment versus a soft-land adjustment in terms of exchange rates, productivity

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and growth?

These are just some of the main questions about the deepening of the US current account deficit. This problem can be read in a larger view if we consider its part in the global imbalances that affect the current account position of other countries, with, in particular, many emerging countries that are running current account surpluses and cumulate US dollar reserves.

Authors like Dooley, Folkerts-Landau and Garber (2004) found an explanation for the willingness of the countries of the periphery running current account surpluses (like Asian countries), to finance the current account deficit of the center country (US). They sketch a revived Bretton Woods system in which the US dollar plays the role of the reserve currency in the system and the countries of periphery, which want to attract Foreign Direct Investments from the United States, accept the role of financing the US deficit because this represents the payment of a collateral for US investments.

In this system, emerging countries deliberately follow a policy of undervaluation of their currency towards the US dollar, so enhancing an export-led growth strategy. At the same time, through the payment of this collateral, periphery countries can have access to the more complete and performing US financial market; therefore capital formation in these emerging countries is not constrained to the more inefficient domestic financial market. This should also be the reason for which periphery countries accept the role of financing the US deficit despite the fact that the payments on US liabilities are not as high as the level of debt would predict. Finally, in the words of Dooley, Folkerts-Landau and Garber, "a chronic US current account deficit is an integral and sustainable feature of a successful international monetary system".

Starting from a different perspective, Gourinchas and Rey (2005) stresses the idea that US act as World Venture Capitalist in the International Monetary System. The authors perform a detailed analysis of the historical evolution of US external assets and liabilities at market value since 1952. They find strong evidence of a sizeable excess return of gross assets over gross liabilities: within each class of assets, the total return (yields and capital gains) that the US has to pay to foreigners is smaller than the total return the US gets on its foreign assets. Along with this finding, Gourinchas and Rey (2005) observe a composition effect according to which US tends to borrow short and lend long, and this effect has been more relevant after the collapse of the Bretton Woods system. This means that the increasing financial globalization allowed US to transform themselves from a World Banker into a World
Venture Capitalist, investing greater amounts in high yield assets such as equity and FDI. The gain US has on the income balance (receipts on assets held less payments on liabilities) represents the exorbitant privilege that the center country enjoys in this international monetary system and allows it to weaken its external constraint. The nature of this exorbitant privilege cannot be clearly defined. It should rely on political power reasons as well as on the market size and the efficiency of the US financial market as well as on the primacy of US dollar as reserve currency.

In a previous paper (2004), Gourinchas and Rey look at the currency composition of international assets and liabilities and argue that capital gains and losses on net foreign assets "constitute an important channel for external adjustment". So asset valuation changes are fundamental in their analysis of the current account deficits or surpluses. In particular, they claim that portfolio revaluations play the main role in the adjustment of the external accounts in the short-medium term while, as everyone should predict, changes in net exports are the source of the adjustment in the long term. Moreover, Gourinchas and Rey (2004) shows that the US can benefit from valuation effects on its deficit as the amount of payments on its external debt is strongly reduced once a depreciation of the dollar occurs. Actually US have almost 95 % of their liabilities in dollar, while on the other hand US have 65 % of assets in foreign currency. So, once there’s one depreciation of the dollar, the amount of payments on liabilities become smaller while the depreciation does not entirely affect the amount of receipts on assets held by US: this means that a depreciation of dollar represents a huge transfer of resources from the rest of the world to US (about 500 billion dollars and a gain of 4 % in terms of GDP).

Lane and Milesi-Ferretti (2006) stresses the importance of the valuation effects in facilitating the global adjustment of current accounts. Moreover the growing international financial integration, measured by the ratio (assets+liabilities/GDP), allows US to weaken its external constraints through a small depreciation of the dollar. The work of Lane and Milesi-Ferretti adds some new evidence on the balance of net foreign position as assets and liabilities are evaluated at their market value rather than at their historical cost as they are usually evaluated in the Balance of Payments.

The valuation effects can therefore give another interesting analysis of the great deficit of the current account of US, while, on the other hand, the economic growth strategy and the accumulation of US dollar reserves by some Asian developing countries can give some explanation on why the adjustment
of the US current account has been delayed until now. Yet, if we want to look at the "fundamentals" of the adjustment we have to consider an increase in exports and a decrease in imports of US. This can be true only through a mechanism of transfer of resources lowering the income of the country in deficit (so reducing its consumption) and raising the income of the rest of the world. In this perspective, Obstfeld and Rogoff (2000) wants to estimate the impact on terms of trade and real exchange rate of a sudden adjustment of the US current account following an unexpected drought of capital flows. The adjustment on the current account means a transfer of resources from US to the rest of the world and, as PPP does not hold for tradable goods, this transfer has an impact on US terms of trade which have to deteriorate. In their following paper, Obstfeld and Rogoff (2004) build a two-country model with a non-tradable good sector and a tradable good sector. The endowments of goods produced are given, so that in the short term production does not change and there’s no factor movement between sectors. Moreover, the presence of a home bias on the consumption of the domestic-produced goods introduces a distortion such that PPP does not hold on tradable goods. The authors put their attention on the change of the price of non-tradable goods that have to fall down even more than the price of tradable goods. Besides, in the panel composition of consumption of households, non-tradable goods account for about 75% of total consumption, so a fall in the price of non-tradable goods causes a great depreciation of RER even more so than the fall in the terms of trade. In order to help the adjustment, the rest of the world should increase the productivity in the non-tradables sector, as faster productivity growth in foreign tradable sector should exacerbate the US current account adjustment. The simulations driven by the authors found that the sudden adjustment should need a depreciation of RER of about 30%, and this gives a confirmation that US adjustment is necessary and deficit no more sustainable. In this view, Roubini (2005) studies two possible scenarios on the US adjustment: a disruptive versus a soft-land adjustment. He claims the need of some policy reforms that increase the level of savings in US and reduce it in Asian countries where currencies should release their peg on dollar. Corsetti, Martin and Pesenti (2007, forthcoming) focus their attention on the transfer of resources that US should give to the rest of the world in order to adjust its current account. In their model, firms produce either for the domestic market or for the foreign market (exports) in an environment of monopolistic competition.
Like in Obstfeld and Rogoff’s articles, they simulate a sudden adjustment of the US current account in order to find the impact that this necessary transfer of resources has on real exchange rate. Alike Obstfeld and Rogoff’s approach, the production is not fixed, there’s factor (labour) movement between sectors and the set of exportables, importables and nontraded goods is endogenous. The authors so stress the importance of the role played by the extensive margin of trade in the adjustment. Strictly speaking, the extensive margin of trade is the number of exporter firms and this model shows that the increase in Home exports is mainly due to the increase in the number of new exporter firms, which is endogenously determined by the free entry conditions on the foreign market.

In Obstfeld and Rogoff (2004), the current account adjustment implies a great depreciation of the real exchange rate: the price of tradable goods falls down so that US tradables become more attractive to foreign households and the price of nontradables falls even further. The adjustment operates only through the channel of the intensive margin of trade: the same firms export more of existing goods as real exchange rate has depreciated.

On the other hand, Corsetti, Martin and Pesenti’s simulations show that, once you take into account the extensive margin of trade, the depreciation needed for the adjustment is smaller than that expected in a model without extensive margin. This paper open the way to a new field of research about the impact of the extensive margin in Macroeconomics adjustment, and gives a new interesting lecture of the transfer problem.

On the same path of Corsetti, Martin, Pesenti (2007), in this paper I will analyse the adjustment of the US current account in a model where the extensive margin of trade is at work with the adjustment.

Alike Corsetti, Martin, Pesenti (2007), my paper introduces firms’ heterogeneity in terms of productivity of labour in a general equilibrium model, which allows me to study the impact of an exchange rate depreciation on the productivity thresholds of exporter and local-oriented firms and then on the average productivities of the two countries. I claim that firms’ heterogeneity allows a new measure of the average productivity of producers that can be considered at a more ”disaggregated” level than the standard measure used by national accounting. Interestingly, this measure is endogenous to the model and depends directly on the shock in the economy, therefore it suggests unexplored consequences of the adjustment of the current account on the average productivity of firms.
2 The adjustment of the US current account in a simple model of firms heterogeneity

2.1 Model framework

In a simple model framework with two countries (H and F) and one tradable sector, each firm produces one variety and pays a variable cost, which depends negatively on its productivity, together with a fixed cost. In this model with only one tradable sector, all firms are potentially exporters.

Free entry conditions on profits made by Home firms on domestic and foreign market will determine two levels of productivity thresholds. These two cut-offs on productivity will endogenously define the number of firms producing for the domestic and the foreign market. Actually, only firms with a level of productivity above the threshold required to be active on foreign market will be exporters. Other firms, with a lower level of productivity, will be active just on the domestic market and finally, least productive firms will be forced to be inactive. This means that this simple model can account for the endogenous non-tradedness of less productive firms and so match the stylized facts on productivity and exports.

Production

Firms in the economy produce the final tradable good for the domestic (D) or the export market (X) by using labour as the only input, wages are taken as the numeraire and equalized to 1. The production function is:

\[ Y(q_i) = \alpha(x) l(q_i) \]

with \( i = D, X \) and \( \frac{\partial \alpha}{\partial x} > 0 \); \( q_i \) represents the quantities of each variety \( i \) of the tradable good that are produced while the resource constraint on labour market is simply:

\[ L = L^H + L^F \]

Firms have access to the same linear technology in labour but the productivity of labour \( x \) that they use as input differs between firms and so generates firms heterogeneity.

I am assuming that the function of distribution of the productivity of labour is a Pareto function, which is a cumulative distribution function of a Pareto
random variable $X$ with the shape parameter $\gamma$:

$$F(x) = 1 - \left(\frac{b}{x}\right)^\gamma, \quad \text{with } x \geq b > 0$$

where $b$ is a scale parameter that bounds the support $[b, +\infty)$ from below. Log $X$ is then distributed exponentially with a standard deviation equal to $1/\gamma$. Any truncation from below of $X$ is also distributed Pareto with the same shape parameter $\gamma$. $X$ has a finite variance if and only if $\gamma > 2$; I therefore assume that $\gamma > \sigma - 1$, which ensures that both the distribution of productivity draws and the distribution of firm sales have finite variances.

So, given the Pareto function, $dF(x) = \gamma b^\gamma x^{-\gamma - 1} dx$.

**FUNCTION COST**

Firms producing for the domestic and foreign market face the same function cost. It consists in two parts: the variable cost which depends negatively on the productivity of labour and increases in the quantity of goods produced, and the fixed cost ($F_i$) which is a given amount of worked hours that firms have to pay in order to enter the local or the export market:

$$\overline{C}(x) = w \left(\frac{q}{x} + F_i\right) \quad \text{with } F_i = \begin{cases} F_D & \text{if } i = D \\ F_X & \text{if } i = X \end{cases}$$

The function cost is the same across the two countries, as I am assuming that the fixed cost of entering the local market ($F_D$), in terms of worked hours, is the same for country H and F as well as the fixed cost of entering the export market ($F_X$).

Moreover, both for Home country and Foreign country, I make the assumption that:

$$F_X = (1 + \delta)F_D \quad \text{with } \delta > 0$$

Parameter $\delta$ reflects the difference between the fixed export cost and the fixed local cost. Actually, when firms export, they face transaction costs greater than those payed when they sell on their local market. This means that exporter firms need more worked hours to pay these additional transaction costs with respect to the fixed cost payed by firms active just on the local market.

The presence of these fixed costs generates a process of autoselection by the firms serving the local or the export market. According to this process, firms
will choose to sell their varieties only in their local market or even choose to export while less productive firms, which cannot pay the fixed costs, are therefore forced to be inactive. Finally, even if there’s only one tradable sector in this economy, the model can account for the endogenous nontradedness by less productive firms and the exit by the least productive ones.

**Households**

\[ U = \log C \]

In the Home country the utility of the representative household is a positive function of consumption \( C \). Consumption for home households is based on a panel of domestic-produced good and imports (foreign export), composed in a CES function, so I have:

\[
U = \log \left[ \int_{\pi_D}^{\infty} q_D(x)^{1-\frac{1}{\sigma}} dF(x) + \int_{\pi_X}^{\infty} q_X^*(x^*)^{1-\frac{1}{\sigma}} dF(x^*) \right]^{\frac{\sigma}{\sigma-1}}
\]

\( \pi_D \) is the threshold on the productivity of domestic firms which are active on the domestic market whereas \( \pi_X \) is the threshold on the productivity of foreign firms who export in Home country. \( \sigma \) denotes the elasticity of substitution across varieties, as well as the elasticity of substitution between import-competing goods \( q_D(x) \) and imports \( q_X^*(x^*) \).

The budget constraint of the representative Home household is:

\[
\int_{\pi_D}^{\infty} p_D(x)c_D(x) dF(x) + \int_{\pi_X}^{\infty} p_X^*(x^*)c_X^*(x^*) dF(x^*) + I \leq l + \frac{\Pi}{L} - \frac{T}{L_H}
\]

Note that \( c_D(x) \) is the consumption of domestic-produced goods while \( c_X^*(x^*) \) is the consumption of foreign-produced goods (imports) which price, \( p_X^* \), is expressed in the Foreign currency; \( I \) is the investment in a well-diversified international portfolio of claims on firm’s profits worldwide.

On the right hand side of the budget constraint, \( l \) represents the income coming from the household’s labour, while \( \frac{\Pi}{L} \) is the share of the world total profits which are equally redistributed to the households in the economy. Finally, the income of each Home household is reduced by an amount equal to \( \frac{1}{T_H} \) of the transfer of resources going from the Home country to the Foreign country (the rest of the world).

**Prices**
In this model all prices are defined in their national currency so I use the term $\epsilon$ to identify the exchange rate between the national and the foreign currency, expressed by home units of labour in terms of foreign ones. The exchange rate $\epsilon$ is therefore equal to the wage ratio between Home and Foreign country. I can therefore write the prices as:

$$p_D(x) = \frac{\sigma}{\sigma - 1} \frac{1}{x} w$$

$$p^*_X(x^*) = \frac{\sigma}{\sigma - 1} \frac{\tau}{x^*} w^*$$

$$p_X(x) = \frac{\sigma}{\sigma - 1} \frac{\tau}{x} w$$

$$p^*_D(x^*) = \frac{\sigma}{\sigma - 1} \frac{1}{x^*} w^*$$

The CES utility function allows me to write the price index for Home country as:

$$P = \left[ \int_{\pi_D}^{\infty} p_D(x)^{1-\sigma} dF(x) + \int_{\pi_X}^{\infty} [\epsilon p^*_X(x^*)]^{1-\sigma} dF(x^*) \right]^{\frac{1}{1-\sigma}}$$

and, by the same means, I derive the price index for the Foreign country as:

$$P^* = \left[ \int_{\pi_D}^{\infty} [\epsilon p^*_D(x^*)]^{1-\sigma} dF(x^*) + \int_{\pi_X}^{\infty} p_X(x)^{1-\sigma} dF(x) \right]^{\frac{1}{1-\sigma}}$$

Note that the two price indexes are both defined in terms of Home currency.

**EXPORTS**

Now I can turn to the analysis of the international trade and I want to look at the profits made by firms who decide to export both in Home and Foreign country.

The value of exports for a firm in Home country is simply: $t^H(x) = p_X(x) q_X(x)$ and the same is true for a firm in the Foreign country. Therefore, I can write the value of exports for Home and Foreign country both expressed in Home currency as:

$$t^H(x) = \left[ 1 + \frac{\Pi}{L} + \frac{T}{L^F} \right] L^F \left[ \frac{p_X(x)}{p^*_X(x^*)} \right]^{1-\sigma}$$

and

$$t^F(x^*) = \left[ 1 + \frac{\Pi}{L} - \frac{T}{L^H} \right] L^H \left[ \frac{\epsilon p^*_X(x^*)}{P} \right]^{1-\sigma}$$
2.2 Free entry conditions and productivity cut-offs

This model provides endogenously the entry and exit of Home firms from both the domestic and foreign market as they choose their export or local-oriented activity by comparing profits on domestic and foreign market. Each firm is a monopolist for the variety it produces and faces free entry conditions, which imply that profits have to be equal to 0.

The following free entry condition applies to Home firms producing for the domestic market:

\[ \pi_D^H = \frac{1}{\sigma} \left[ 1 + \frac{\Pi}{L} - \frac{T_L}{L^H} \right] L^H \left( \frac{p_D(x)}{P} \right)^{1-\sigma} - F_D = 0 \]

As wages are the numeraire, \( F_D \) represents the value of worked hours paid by Home firms entering the domestic market. This value is expressed in terms of Home currency, as well as all the profits in the free entry conditions. Then, once I substitute for the price of domestic-produced goods, I find:

\[ \bar{x}_D = \mu_2 \left( \frac{F_D}{L^H} \right)^{\frac{1}{1-\sigma}} \frac{1}{\bar{P}} \]  

(1)

with

\[ \mu_2 = \sigma^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} \left[ 1 + \frac{\Pi}{L} - \frac{T_L}{L^H} \right]^{-\frac{1}{\sigma - 1}} \]

\( \bar{x}_D \) is the Home local market cut-off of productivity, obtained for given price index \( P \). Free entry condition tells us that Home firms with a level of productivity above \( \bar{x}_D \) are active on the local market and produce import-competing goods, while the others are forced to exit.

The same free entry condition applies then on firms in the Foreign country\(^1\):

\[ \pi_D^F = \frac{1}{\sigma} \left[ 1 + \frac{\Pi}{L} + \frac{T_L}{L^F} \right] L^F \left( \frac{\epsilon p_D^*(x^*)}{P^*} \right)^{1-\sigma} - \epsilon F_D = 0 \]

so that I can found the following threshold on the labour productivity of the Foreign firms producing for their local market:

\[ \bar{x}_D^* = \mu_1 \left( \frac{F_D}{L^F} \right)^{\frac{1}{\sigma - 1}} \frac{1}{P^* \epsilon^{\frac{\sigma}{\sigma - 1}}} \]  

(2)

\(^1\)Once again note that also the profits of Foreign local-oriented firms are expressed in the Home currency and \( \epsilon F_D \) is the value, expressed in terms of Home currency, of the worked hours paid by the Foreign firms entering their local market.
with
\[
\mu_1 = \sigma \frac{\sigma - 1}{\sigma - 1} \left[ 1 + \frac{\Pi}{L} + \frac{T}{L^F} \right]^{-\frac{1}{\sigma - 1}}
\]
Now if I look at the exporter firms, I can easily define the profits made by the Home exporter firm as:
\[
\pi_{exp}^H = \frac{1}{\sigma} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \left[ 1 + \frac{\Pi}{L} + \frac{T}{L^F} \right] L^F \left[ \frac{x^{-1}F}{P^*} \right]^{1-\sigma} - (1 + \delta)F_D
\]
Once again the free entry condition tells us that \( \pi_{exp}^H = 0 \) so I can find the critical threshold of labour productivity for the Home exporter firm:
\[
\bar{x}_{exp} = \mu_1 \left[ \frac{(1 + \delta)F_D}{L^F} \right]^{\frac{1}{\sigma - 1}} \frac{\tau}{P^*}
\] (3)
All Home firms with a level of productivity above \( \bar{x}_{exp} \) are exporters on the Foreign market, whereas firms with a level of productivity \( \bar{x}_D \leq x < \bar{x}_{exp} \) are active just on the domestic market, and finally, least productive firms are forced to be inactive.
By the same means, I can write the value of profits for the Foreign exporter firm expressed in the home currency as:
\[
\pi_{exp}^F = \frac{1}{\sigma} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \left[ 1 + \frac{\Pi}{L} - \frac{T}{L^H} \right] L^H \left[ \frac{x^{\epsilon,1}F}{P} \right]^{1-\sigma} - \epsilon(1 + \delta)F_D
\]
FEC on the export market for Foreign firms gives me the critical threshold of labour productivity for the firms entering the domestic country, so that \( \bar{x}_{exp}^* \) represents the productivity cut-off for Home imports:
\[
\bar{x}_{exp}^* = \mu_2 \left[ \frac{(1 + \delta)F_D}{L^H} \right]^{\frac{1}{\sigma - 1}} \frac{\tau}{P^*} \epsilon^{\frac{\sigma}{\sigma - 1}}
\] (4)

2.3 Equilibrium price indexes
Equations (1),(2),(3) and (4) represent the thresholds on productivity obtained for given prices. The equilibrium price indexes in the Home and Foreign markets will depend on the firms operating in that country and, as firms are heterogeneous in the model, their entry and exit on Home and Foreign markets imply that the equilibrium price index of a country depends on
the productivity of active firms. Therefore, I take the Home price index and I substitute prices of varieties and resolve integrals to obtain:

\[ P = \left[ \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{\gamma b^\gamma}{\gamma - \sigma + 1} \bar{x}D^{\sigma - \gamma - 1} + \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{\gamma b^\gamma}{\gamma - \sigma + 1} \bar{x}_e^{\sigma - \gamma - 1} \right]^{\frac{1}{1-\sigma}} \]

then I plug in the thresholds of the equations (1) and (4), and after some algebraic manipulations I have the equilibrium price index for the Home country\(^2\):

\[ P = \left[ \mu_3 \left( \frac{F_D}{L_H} \right)^{\frac{\sigma - \gamma - 1}{\gamma - 1}} \left( 1 + A \tau^{-\gamma} \epsilon \frac{\sigma - \gamma - 1}{\sigma - 1} \right) \right]^{-\frac{1}{\gamma}} \]  

(5)

where A is a constant equal to \( A = (1 + \delta) \frac{\sigma - \gamma - 1}{\sigma - 1} \). In the Foreign country, the price index is:

\[ P^* = \left[ \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{\gamma b^\gamma}{\gamma - \sigma + 1} \bar{x}D^{\sigma - \gamma - 1} + \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{\gamma b^\gamma}{\gamma - \sigma + 1} \bar{x}_e^{\sigma - \gamma - 1} \right]^{\frac{1}{1-\sigma}} \]

here I have to plug in the productivity threshold of Foreign local-oriented firms (equation 2) as well as the cut-off of Home exporter firms (equation 3) who are entering the Foreign market to sell varieties, which prices will have a weight in the Foreign price index. So I get the resulting equation\(^3\):

\[ P^* = \left[ \mu_4 \left( \frac{F_D}{L_F} \right)^{\frac{\sigma - \gamma - 1}{\gamma - 1}} \left( \epsilon \frac{\sigma - \gamma - 1}{\sigma - 1} \frac{\sigma - \gamma - 1}{\sigma - 1} + A \tau^{-\gamma} \right) \right]^{-\frac{1}{\gamma}} \]  

(6)

Equilibrium price indexes of Home and Foreign country both depend linearly on the transport iceberg cost \( \tau \) as well as on the exogenous fixed cost \( F_D \) and on the transfer of resources \( T \) coming from Home toward the Foreign country.

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\(^2\)Where \( \mu_3 \) is a constant equal to \( \mu_3 = \left( \frac{\sigma}{\sigma - 1} \right)^{\gamma} \frac{\gamma b^\gamma}{\gamma - \sigma + 1} \frac{\sigma - \gamma - 1}{\sigma - 1} \left[ 1 + \frac{\Pi}{L} - \frac{T}{L} \right]^{-\frac{\sigma - \gamma - 1}{\sigma - 1}} \).

I am actually taking total profits \( \Pi \) as a constant too, and I will prove later that \( \Pi \) isn’t affected by the transfer \( T \).

\(^3\)Where \( \mu_4 \) is a constant equal to \( \mu_4 = \left( \frac{\sigma}{\sigma - 1} \right)^{\gamma} \frac{\gamma b^\gamma}{\gamma - \sigma + 1} \frac{\sigma - \gamma - 1}{\sigma - 1} \left[ 1 + \frac{\Pi}{L} + \frac{T}{L} \right]^{-\frac{\sigma - \gamma - 1}{\sigma - 1}} \).
2.4 Equilibrium thresholds and exchange rate

Free entry conditions on Foreign and Home market give me the critical thresholds on productivity for local-oriented and exporter firms operating on both countries. These thresholds are obtained for given price indexes but I can now find the equilibrium value of these thresholds just by plugging them into the equilibrium price indexes of Home and Foreign country. As a result I will obtain an expression for each productivity cut-off which will depend only on the endogenous exchange rate $\epsilon$ and on the exogenous transport cost $\tau$ and transfer $T$.

Firstly, I substitute the Home price index in equations (1) and (4) and I have:

$$\bar{x}_D = \mu_2 \left( \frac{F_D}{L_H} \right) \frac{1}{\sigma - 1} \left[ \mu_3 \left( \frac{F_D}{L_H} \right)^{\frac{\sigma - \gamma - 1}{\sigma - 1}} (1 + A\tau^{-\gamma} \epsilon^{\frac{\sigma - \gamma - 1}{\sigma - 1}}) \right]^{\frac{1}{\gamma}}$$

(7)

Then, I plug the equilibrium Foreign price index into equations (2) and (3) so that:

$$\bar{x}_D = \mu_1 \left( \frac{F_D}{L_F} \right) \frac{1}{\sigma - 1} \epsilon^{\frac{\sigma}{\sigma - 1}} \left[ \mu_4 \left( \frac{F_D}{L_F} \right)^{\frac{\sigma - \gamma - 1}{\sigma - 1}} \left( \epsilon^{\frac{\sigma - \gamma - 1}{\sigma - 1}} + A\tau^{-\gamma} \right) \right]^{\frac{1}{\gamma}}$$

(9)

The levels of productivity thresholds for both exporter and local-oriented firms depend on the price index of the market where they are active and on the transfer. It is useful here to total differentiate equations (7)-(10) in order to have more indications of the way the transfer directly affects the thresholds: by lowering Home income and raising Foreign income, as well as indirectly through the change in the exchange rate. Note that I total differentiate the productivity thresholds in the neighborhood of the symmetrical equilibrium with $\epsilon = 1$ and $T = 0$ so that I have:

$$d\bar{x}_D = \frac{1}{\sigma - 1} \frac{1}{L_H} \left[ 1 + \Pi \right]^{-1} \mu_2 \left( \frac{F_D}{L_H} \right) \frac{1}{\sigma - 1} P^{-1}dT + \mu_2 \left( \frac{F_D}{L_H} \right)^{\frac{1}{\gamma}} \frac{1}{\gamma} \left[ \ldots \right]^{-1} P^{-1}d\left[ \ldots \right]$$
I know that \( \bar{x}_D = \mu \left( \frac{F_D}{L^H} \right)^{\frac{1}{\sigma - 1}} P^{-1} \) and \( \frac{dP}{P} = \frac{1}{\gamma} \left[ \ldots \right]^{-1} d \left[ \ldots \right] \) and using the same substitutions on the other thresholds and Foreign price index I can easily total derivate all cut-offs as:

\[
\frac{d\bar{x}_D}{\bar{x}_D} = \frac{1}{\sigma - 1} \left[ 1 + \frac{\Pi}{L} \right]^{-1} dT - \frac{dP}{P}
\]

\[
\frac{d\bar{x}_{exp}^*}{\bar{x}_{exp}^*} = \frac{1}{\sigma - 1} \left[ 1 + \frac{\Pi}{L} \right]^{-1} dT + \frac{\sigma}{\sigma - 1} d\epsilon - \frac{dP^*}{P^*}
\]

\[
\frac{d\bar{x}_D}{\bar{x}_D} = -\frac{1}{\sigma - 1} \left[ 1 + \frac{\Pi}{L} \right]^{-1} dT + \frac{\sigma}{\sigma - 1} d\epsilon - \frac{dP^*}{P^*}
\]

\[
\frac{d\bar{x}_{exp}}{\bar{x}_{exp}} = -\frac{1}{\sigma - 1} \left[ 1 + \frac{\Pi}{L} \right]^{-1} dT - \frac{dP^*}{P^*}
\]

The productivity thresholds of firms operating in the Home market, both the Home local-oriented firms and the Foreign exporters, have to go up once the transfer is gone, while on the other hand, they fall down in the Foreign market. A growth in \( \epsilon \) raises the productivity cut-off both of Foreign exporters and the productivity threshold of Foreign local-oriented firms. Finally, the price index on the market where firms are active always has a negative effect on cut-offs as a lower price index means greater competition and therefore firms have to be more productive to enter this market, so raising the required threshold.

Moreover \( \frac{dP}{P} \), as well as \( \frac{dP^*}{P^*} \), is function of \( d\epsilon \) and \( dT \) and in general equilibrium, the differentiation of the balanced current account will give me the equilibrium relation \( \frac{d\epsilon}{dT} \) which is very important to disentangle the direct effect of \( T \) on thresholds from its indirect effect through the exchange rate.

### 2.5 The balanced current account at the symmetrical equilibrium

The balanced current account at equilibrium is such that:

\[
EXP - IMP - T = 0
\]
so I can write the value of Home exports and imports (sales) both expressed in Home currency as follows:

\[ EXP = \int_{\bar{x}_{exp}}^{\infty} \left( \frac{\sigma - 1}{\sigma} \right)^{\gamma - 1} \left[ 1 + \frac{\Pi}{L} + \frac{T}{LF} \right] L^F \left[ \frac{x_{exp}^{1-\gamma}}{\bar{P}^{\gamma}} \right]^{1-\gamma} dF(x) \]

\[ IMP = \int_{\bar{x}_{exp}}^{\infty} \left( \frac{\sigma - 1}{\sigma} \right)^{\gamma - 1} \left[ 1 + \frac{\Pi}{L} - \frac{T}{L^H} \right] L^H \left[ \frac{x_{exp}^{1-\gamma}}{\bar{P}^{\gamma}} \right]^{1-\gamma} dF(x^*) \]

Now, I do integrals and evaluate them for the equilibrium levels of cut-offs in equations (7)-(10), so that I can obtain EXP and IMP depending on the price indexes of Home and Foreign country, on the transfer and on the exchange rate \( \epsilon \):

\[ EXP = \lambda_1 \left[ 1 + \frac{\Pi}{L} + \frac{T}{LF} \right] \tau^{-\gamma} P^{\gamma} \mu_1^{\gamma-1} A \left( \frac{F_D}{LF} \right)^{\frac{\gamma-1}{\sigma-1}} \]

\[ IMP = \lambda_2 \left[ 1 + \frac{\Pi}{L} - \frac{T}{L^H} \right] \epsilon^{\frac{\sigma-\gamma-1}{\sigma-1}} \tau^{-\gamma} P^{\gamma} \mu_2^{\gamma-1} A \left( \frac{F_D}{L^H} \right)^{\frac{\gamma-1}{\sigma-1}} \]

I want to find the impact that the transfer has on exports and imports and on the exchange rate once the adjustment of the current account is at work. Therefore, I total differentiate EXP and IMP in the neighborhood of the symmetrical equilibrium where \( T = 0, \epsilon = 1 \) and \( L^H = LF \).

**Symmetrical equilibrium**

Remember that, at the symmetrical equilibrium, exports have to be equal to imports and I find that:

\[ EXP = IMP = \bar{\lambda} \left[ 1 + \frac{\Pi}{L} \right] \tau^{-\gamma} \mu_1^{\sigma-1} A \left( \frac{2F_D}{L} \right)^{\gamma-1} \bar{P}^{\gamma} \]

At this point, it is useful to remark that when countries are perfectly symmetrical the price index in Home and Foreign country is also the same. Taking

\[ 4 \text{Where } \lambda_1 \text{ is a constant equal to } \lambda_1 = \left( \frac{\sigma - 1}{\sigma} \right)^{\gamma-1} \frac{b^\gamma}{\gamma - \sigma + 1} LF \text{ and } \lambda_2 = \left( \frac{\sigma - 1}{\sigma} \right)^{\gamma-1} \frac{b^\gamma}{\gamma - \sigma + 1} L^H \]

\[ 5 \text{Note that at the symmetrical equilibrium } T=0 \text{ and therefore } \mu_1 = \mu_1 = \mu_2 \text{ and also } \bar{\lambda} = \lambda_1 = \lambda_2 \text{ as well as } \bar{\mu} = \mu_3 = \mu_4. \]
equations 5 and 6 I obtain:

\[ \bar{P} = \left[ \bar{\mu} \left( \frac{2F_D}{L} \right)^{\frac{\sigma - \gamma - 1}{\sigma - 1}} \left( 1 + A\tau^{-\gamma} \right) \right]^{-\frac{1}{\gamma}} \]

Consequently, once I total differentiate Home and Foreign price indexes in the neighborhood of the symmetric equilibrium I easily observe that:

\[ \frac{dP^*}{P} = -\frac{1}{\gamma} \left[ \frac{-\sigma - \gamma - 1}{\sigma - 1} \left( 1 + \frac{\Pi}{L} \right)^{-1} dT + \frac{1}{1 + A\tau^{-\gamma}} \frac{\sigma - \gamma\sigma - 1}{\sigma - 1} d\epsilon \right] \]

and

\[ \frac{dP}{P} = -\frac{1}{\gamma} \left[ \frac{\sigma - \gamma - 1}{\sigma - 1} \left( 1 + \frac{\Pi}{L} \right)^{-1} dT + \frac{A\tau^{-\gamma}}{1 + A\tau^{-\gamma}} \frac{\sigma - \gamma\sigma - 1}{\sigma - 1} d\epsilon \right] \]

**The total differentiation of the current account**

I start by writing the total differential of exports in the neighborhood of the symmetrical equilibrium as:

\[ \frac{dEXP}{EXP} = \left( 1 + \frac{\Pi}{L} \right)^{-1} \frac{2}{L} \frac{\gamma}{\sigma - 1} dT + \gamma \frac{dP^*}{P} \]

Now I can substitute the differential of the Foreign price index into the total differential of exports, so that:

\[ \frac{dEXP}{EXP} = \left( 1 + \frac{\Pi}{L} \right)^{-1} \frac{2}{L} \frac{\gamma}{\sigma - 1} dT - \frac{1}{1 + A\tau^{-\gamma}} \frac{\sigma - \gamma\sigma - 1}{\sigma - 1} d\epsilon \]  (11)

Then, I look at imports and total differentiate IMP to get:

\[ \frac{dIMP}{IMP} = \left( 1 + \frac{\Pi}{L} \right)^{-1} \frac{2}{L} \frac{\gamma}{\sigma - 1} dT + \frac{\sigma - \gamma\sigma - 1}{\sigma - 1} d\epsilon + \gamma \frac{dP}{P} \]

After the substitution of the total differential on the Home price index into the total differential of imports I have:

\[ \frac{dIMP}{IMP} = -\left( 1 + \frac{\Pi}{L} \right)^{-1} \frac{2}{L} dT + \frac{1}{1 + A\tau^{-\gamma}} \frac{\sigma - \gamma\sigma - 1}{\sigma - 1} d\epsilon \]  (12)

Home country exports depend positively on the transfer and on the exchange rate \( \epsilon \). In fact, there’s a direct positive effect of the transfer on the income
of the Foreign country, so that Foreign demand raises up and Home exports go up too. On the other hand, if the exchange rate grows up, it makes Home exports cheaper and so more attractive to Foreign households who will demand more of them. Actually, this is only a part of history as the change in the exchange rate is endogenous in this model and depends on the transfer of resources itself, and the total differentiation of the balanced current account will show this effect at symmetrical equilibrium.

The differentiation of exports and imports shows that they only depend on the variation of the exchange rate and the transfer, and only world total profits are to be determined. Until this point, I have considered aggregate world income as the sum of the remuneration of the exogenous total amount of labour and the total profits $\Pi$ which are redistributed to households. Each of them receives dividends revenue from the firms they own and the main hypothesis here is that each household owns an equal share of a well-diversified international portfolio of claims on firm’s profits. The investment made by households finances the fixed costs payed by local-oriented and exporter firms. I can now determine the value of total world profits as:

$$\Pi = \left(\int_{\bar{x}_D}^{\infty} \pi_D^H dF(x) - \int_{\bar{x}_D}^{\infty} F_D dF(x)\right) + \left(\int_{\bar{x}_{exp}}^{\infty} \pi_{exp}^H dF(x) - \int_{\bar{x}_{exp}}^{\infty} (1 + \delta)F_D dF(x)\right) +$$

$$+ \left(\int_{\bar{x}_D}^{\infty} \pi_D^{exp} dF(x) - \int_{\bar{x}_D}^{\infty} \epsilon F_D dF(x^*)\right) + \left(\int_{\bar{x}_{exp}}^{\infty} \pi_{exp}^{exp} dF(x) - \int_{\bar{x}_{exp}}^{\infty} \epsilon(1 + \delta)F_D dF(x^*)\right)$$

Total world profits don’t depend on the transfer$^6$ and $\Pi$ is therefore a positive constant equal to:

$$\Pi = \frac{\sigma - 1}{\gamma \sigma - \sigma + 1} L$$

Now the equilibrium of the model is completely characterized and I can therefore write the balanced current account as:

$$\frac{dEXP}{EXP} \frac{1}{dT} - \frac{dIMP}{IMP} \frac{1}{dT} - \frac{1}{EXP} = 0 \quad (13)$$

Equation (13) will give me the equilibrium value of $\frac{d\epsilon}{dT}$ which balances the current account after I put a shock on it through the introduction of a very

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$^6$This is due to the Pareto function of distribution of firms’ productivity as the transfer affects the profits of some firms but the extra-profits made by some are exactly equal to the losses of others.
small transfer in the direction of the Foreign country. So I will detect the impact of the transfer on the exchange rate, that is the wage ratio between the Home and Foreign country, when countries are perfectly symmetric. Resolving equation (13) I get:

\[
\frac{d\epsilon}{dT} = \left(1 + \frac{\Pi}{I}\right)^{-1} \frac{2}{L} \left(\frac{A\tau^{-\gamma} - 1}{A\tau^{-\gamma}}\right) > 0
\]

3 Implications of the current account adjustment

3.1 Productivity thresholds and exchange rates

The aim of this paper is to find the impact of the transfer on all the endogenous variables in the model, and therefore I will define all variables as \( \hat{a} = \frac{da/a}{dT/L} \). These variables are the domestic and exporter thresholds for Home and Foreign country, the exchange rate \( \epsilon \) which is also the ratio between the Home and Foreign wage, the RER which is the real welfare-based exchange rate, and finally the aggregate consumption \( C \) and the welfare of Home country. Table 1 reports these variables and shows the impact that the transfer has on them at the symmetrical equilibrium.

The total differentiation of the current account where \( \epsilon = 1, T = 0 \) and \( L^H = L^F \) tells us that a marginal variation by the transfer will reduce the required cut-off on the productivity of Home exporter firms, there will therefore be entry of Home firms in the export activity while the cut-off on Home local-oriented firms goes up and so less productive Home firms that were active on the domestic market will be forced to exit.

On the Foreign country the presence of the transfer raises the level of productivity needed to export versus Home country so only the more productive firms will continue to export while other firms will be forced to continue their activity on their local market. On the other hand, transfer lowers the cut-off on the productivity of local-oriented Foreign firms, this means that there will be entry of less productive Foreign firms on the Foreign market.

Why do these changes in cut-offs happen? The transfer of resources has a

\footnote{Note that \( A\tau^{-\gamma} - 1 < 0 \) and \( \sigma - \gamma\sigma - 1 < 0 \), therefore the transfer implies an increase in \( \epsilon \), that is a depreciation of the exchange rate.}
\[ \hat{x}_D = \frac{1}{\gamma} \left( 1 + \frac{\Pi}{L} \right)^{-1} \left[ \frac{1}{2} (A \tau - \gamma + 1) \right] > 0 \]

\[ \hat{x}_{\text{exp}} = -\frac{1}{\gamma} \left( 1 + \frac{\Pi}{L} \right)^{-1} \left[ \frac{1}{2} \left( \frac{1 + A \tau - \gamma}{A \tau - \gamma} \right) \right] < 0 \]

\[ \hat{x}_D^* = -\frac{1}{\gamma} \left( 1 + \frac{\Pi}{L} \right)^{-1} \left[ \frac{1}{2} \left( \frac{1 + A \tau - \gamma}{A \tau - \gamma} \right) \right] + \frac{\sigma (1 + A \tau - \gamma)}{\sigma - \gamma - 1} \left( 1 + \frac{\Pi}{L} \right)^{-1} \left[ \frac{1}{2} \left( \frac{A \tau - \gamma - 1}{A \tau - \gamma} \right) \right] < 0 \]

\[ \hat{x}_{\text{exp}}^* = \frac{1}{\gamma} \left( 1 + \frac{\Pi}{L} \right)^{-1} \left[ \frac{1}{2} (A \tau - \gamma + 1) \right] + \frac{\sigma (1 + A \tau - \gamma)}{\sigma - \gamma - 1} \left( 1 + \frac{\Pi}{L} \right)^{-1} \left[ \frac{1}{2} \left( \frac{A \tau - \gamma - 1}{A \tau - \gamma} \right) \right] > 0 \]

\[ \hat{\epsilon} = \frac{(1+\frac{\Pi}{L})^{-1} \left( \frac{A \tau - \gamma - 1}{A \tau - \gamma} \right)}{\frac{2}{\sigma - \gamma - 1}} > 0 \]

\[ \hat{RER} = -\frac{1}{\gamma} \left[ \frac{2(\sigma - \gamma - 1)}{\sigma - 1} \left( 1 + \frac{\Pi}{L} \right)^{-1} + \frac{1 - A \tau - \gamma}{2} \left( 1 + \frac{\Pi}{L} \right)^{-1} \left( \frac{A \tau - \gamma - 1}{A \tau - \gamma} \right) \right] \]

\[ \hat{C} = \frac{1}{\gamma} \left[ \left( 1 + \frac{\Pi}{L} \right)^{-1} \left( \frac{\sigma - \gamma - 1}{\sigma - 1} + \frac{1}{2} (A \tau - \gamma - 1) \right) \right] - 1 < 0 \]

\[ \hat{U} = \frac{\hat{C}}{\log C} < 0 \]

Table 1: Summary of main results

direct impact on the income of the Foreign country which is raised by T. For given exogenous supply of labour, the higher demand in Foreign country will raise the prices of Home exports and allow less productive Home firms to pay the fixed export cost and enter the foreign market. When we look at the change of the productivity cut-off we can see the extensive margin at work as, in this case, some new firms start to export and therefore they increase the value of aggregate exports. Existing exporters will continue to export more of their varieties, such that also the intensive margin contribute to the growth of exports. At the general equilibrium the transfer of resources will also determine the raise of the exchange rate \( \epsilon \), but, with respect to the case with only intensive margin, the depreciation of the exchange rate is dampened by the presence of the extensive margin. The increased income
of Foreign households will also allow some Foreign firms to pay their entry cost on the Foreign market so that there will be entry of local-oriented firms which were inactive before.

On the other hand, the transfer has reduced the income of the Home country so that it’s more difficult for Home local-oriented firms to remain active on the Home country; the cut-off on productivity therefore is raised by the transfer and some less productive firms are forced to exit. The lower demand by Home households affects Foreign exporter firms too and moreover, the exchange rate has depreciated so Foreign exports are too expensive for Home households. Consequently, the cut-off on the productivity of Foreign exporters has to go up as only a few very productive firms can now pay the fixed cost, which is also augmented by the high $\epsilon$, and entry the Home market.

Finally, in the new general equilibrium, the depreciation of the exchange rate has raised the price index of Home country, although the entry of more productive local-oriented, as imports are much too expensive. In Foreign country the price index (when expressed in terms of Foreign currency) is reduced by the depreciation as Home exports are far cheaper now and also the level of competition has strengthened significantly after the entry of new Home exporters and Foreign local-oriented firms.

### 3.2 GDP and average productivity

Until here, in this economy, the GDP for each country is equal to the GDP issued by the tradable sector.

**Factors’ remuneration**

Let’s define GDP as remuneration of the factors of production. Each worker receives a wage equal to $w$ (remember that $w = 1$ as wage is the numeraire) and some income coming from the share on total profits that he holds:

$$GDP^H = \left[w + \frac{\Pi}{L}\right] L^H$$

and in symmetrical equilibrium $GDP^H = GDP^F$ so that:
GDP = \frac{1}{2} (L + \Pi)

\textbf{Supply side}

From the supply side, GDP is the sum of the value of production for the domestic market (domestic sales) and the value of sales abroad (country H exports). I have therefore:

\begin{align*}
GDP_H &= \int_{\bar{x}_D}^{\infty} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \left[ 1 + \frac{\Pi}{L} - \frac{T}{L^H} \right] L^H \left[ \frac{\bar{x}_D}{P} \right]^{1-\sigma} dF(x) + \\
&+ \int_{\bar{x}_{exp}}^{\infty} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \left[ 1 + \frac{\Pi}{L} + \frac{T}{L^F} \right] L^F \left[ \frac{\bar{x}_{exp}}{P^*} \right]^{1-\sigma} dF(x)
\end{align*}

Now, resolving integrals for the equilibrium value of the productivity thresholds, I obtain:

\begin{align*}
GDP_H &= \lambda_2 \left[ 1 + \frac{\Pi}{L} - \frac{T}{L^H} \right] P^\gamma \mu_2^{\sigma - \gamma - 1} \left( \frac{F_D}{L^H} \right)^{\frac{\sigma - 1}{\sigma - 1}} + \\
&+ \lambda_1 \left[ 1 + \frac{\Pi}{L} + \frac{T}{L^F} \right] P^\gamma \mu_1^{\sigma - \gamma - 1} \left[ 2 \frac{F_D}{L} \right]^{\frac{\sigma - 1}{\sigma - 1}} (1 + A \tau - \gamma)
\end{align*}

As for the remuneration of factors, I want to find the GDP of home country (the same for the foreign country) at symmetrical equilibrium, where \( L^H = L^F \) and \( T = 0 \) so that \( \epsilon = 1 \). At symmetrical equilibrium \( GDP_H = GDP_F \) and:

\[ \bar{GDP} = \bar{\lambda} \left[ 1 + \frac{\Pi}{L} \right] P^\gamma \bar{\mu}_1^{\sigma - \gamma - 1} \left( 2 \frac{F_D}{L} \right)^{\frac{\sigma - 1}{\sigma - 1}} (1 + A \tau - \gamma) \]

Now, just substitute the value of the parameters \( \bar{\lambda} \equiv \lambda_1 = \lambda_2 \) and \( \bar{\mu}_1 \equiv \mu_1 = \mu_2 \) as well as \( \bar{\mu} \equiv \mu_3 = \mu_4 \) and the symmetrical equilibrium price index \( \bar{P} \):

\[ \bar{P} = \left[ \bar{\mu} \left( 2 \frac{F_D}{L} \right)^{\frac{\sigma - 1}{\sigma - 1}} (1 + A \tau - \gamma) \right]^{-\frac{1}{\gamma}} \]
and find that

$$\overline{GDP} = \frac{1}{2} (L + \Pi)$$

This result proves that the model is well settled, as at general equilibrium GDP is the same from factors’ remuneration and supply side.

**Average productivity**

According to the standard national accounting definition, I can now define the productivity of labour (the only factor in this economy) as:

$$\text{productivity} \equiv \frac{GDP^H}{L^H} = 1 + \frac{\Pi}{L}$$

This is a general aggregate measure of productivity, which doesn’t take into account the impact of the heterogeneity in the productivity of firms.

Here I want to look for a measure of the average productivity in the tradable sector which accounts for the distribution of productivity. Moreover, the definition of the productivity thresholds allows me to distinguish firms who are producing just for the domestic market from those one who are exporting their products.

My measure of the overall average productivity in the tradable sector is a weighted mean of the productivity of domestic-oriented firms and exporters. If I consider the productivity of the domestic-oriented firms, according to the Pareto function of distribution of the productivity, I will find out that the mean of the productivity of these firms is:

$$\frac{1}{1 - F(\bar{x}_D)} \int_{\bar{x}_D}^{\infty} x \, dF(x) = \frac{\gamma}{\gamma - 1} \bar{x}_D$$

while the mean of the productivity of Home exporters is equal to:

$$\frac{1}{1 - F(\bar{x}_{exp})} \int_{\bar{x}_{exp}}^{\infty} x \, dF(x) = \frac{\gamma}{\gamma - 1} \bar{x}_{exp}$$

On the other hand, I want to weight the productivity of the domestic-oriented firms and exporters by the sales that they realise on the Home
market (for domestic-oriented) and the Foreign market (for exporters). Actually, firms’ heterogeneity ensures an endogenous separation between the domestic-oriented and the exporter firms, and the amounts of sales of these two endogenously created "subsectors" of the tradable sector depend on the conditions on the Home and Foreign markets, specifically on the income of households and on price indexes.

We know that the shock in this economy, that is the transfer of resources going from country H to country F, has an impact on the income of households and price indexes while $GDP^H$ is not affected as $T$ raises the demand of exporters and decreases the demand for domestic firms and moreover $GDP^H = L^H \left( 1 + \frac{H}{L} \right)$ where II is a constant too. Therefore, the share of $GDP^H$ deriving from the sales on the domestic market as well as the share of $GDP^H$ arising from the sales of exporter firms abroad will both vary with the transfer. In particular, the ratio of export sales over the total $GDP^H$ increases as the real adjustment of the current account needs an increase in exports while the ratio of the domestic sales over $GDP^H$ will be reduced.

For this reason, I take the market shares of the domestic firms and exporters as weights so that the average productivity can be defined as follows:

$$\tilde{x} = \frac{DOM \ SALES \left[ \frac{1}{\gamma - 1} \tilde{x}_D \right] + EXP \ SALES \left[ \frac{1}{\gamma - 1} \tilde{x}_{exp} \right]}{GDP^H}$$

Note that both weights, the domestic sales and export sales, are expressed in value and sum together to the national GDP, expressed in value as well. Therefore, the measure of average productivity that I obtain refers exactly to the units of Labour that are needed in average to produce one unit of good in the Home country. As both the numerator and the denominator of $\tilde{x}$ are defined in value, I'm implicitly deflating this measure by the price index at production and I can be sure that the level of $\tilde{x}$ is purged of the effects of the transfer on the price of domestic produced goods and export goods. Once again, note that the price index that I am considering into the $GDP^H$ refers to the price index of Home producers, which differs from the consumer-based price index which depends on the price of domestic-produced goods and imports.

At the symmetrical equilibrium, I obtain:

$$\hat{x} = \frac{\gamma}{\gamma - 1} \left[ \frac{1}{1 + A \tau^{-\gamma}} \tilde{x}_D + \frac{A \tau^{-\gamma}}{1 + A \tau^{-\gamma}} \tilde{x}_{exp} \right]$$
The average productivity $\hat{x}$ is a function of the level of the domestic-oriented productivity threshold as well as the level of exporter threshold. If I totally differentiate $\hat{x}$ around the symmetrical equilibrium, I can define $\hat{x} = \frac{dx}{dT/L}$ as for the other variables in section 3.1 and I have:

$$\hat{x} = \frac{\gamma}{\gamma - 1} \left( \frac{1}{1 + A^{T - \gamma}} \right) \hat{x}_D + \frac{\gamma}{\gamma - 1} \left( \frac{A^{T - \gamma}}{1 + A^{T - \gamma}} \right) \hat{x}_{exp} > 0$$

This is a crucial result because, at the symmetrical equilibrium, an even very small transfer of resources will increase the productivity threshold of the domestic-oriented firms ($\hat{x}_D > 0$) while reducing the threshold of the exporter firms ($\hat{x}_{exp} < 0$), as it is showed in table 1. It is therefore important to study such a measure of average productivity as it depends on the change in productivity thresholds, and these cut-offs don’t go in the same direction once the transfer is set up.

Nevertheless, I can conclude that the average productivity will increase as a consequence of the transfer as $A^{T - \gamma} < 1$. In other words, the share of domestic sales over the total GDP is reduced by the transfer but stays bigger than the share of export sales in absolute terms. This is due to the presence of the greater fixed costs ($\delta$) that firms have to pay to export as well as the transport iceberg cost $\tau$.

4 Quantitative simulations

In this section I make some quantitative simulations on the model in order to find the percentage variation of the real exchange rate to a transfer of resources from the Home country, here US and the Foreign country, that is the rest of the world. First of all, I normalize the size of world Labour $L = 200$ and set $L^H$ such that the GDP of Home country is equal to 27% of world GDP as data report for US in 2006. Then, I set the amount of the transfer such that T is equal to 7% of Home country GDP: this transfer is equal to the present deficit of the current account of US and therefore it is that one required to suddenly balance the CA. Remark that GDP depends on the level of $\gamma$ and $\sigma$ chosen, therefore also the amount of T will vary according to the change of the parameters.

The following Table 2 reports the main results of my numerical simulations.
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<th></th>
<th>$\bar{x}_D$</th>
<th>$\bar{x}_{exp}$</th>
<th>$\bar{x}^*_D$</th>
<th>$\bar{x}^*_{exp}$</th>
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<th>$\epsilon$</th>
<th>RER</th>
<th>st.dev.</th>
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<td>8.84</td>
<td>1.12</td>
<td>6.70</td>
<td>5.79</td>
<td>0.005</td>
</tr>
<tr>
<td>$\gamma = 30$</td>
<td>0.24</td>
<td>-16.18</td>
<td>0.48</td>
<td>20.17</td>
<td>0.89</td>
<td>15.61</td>
<td>16.69</td>
<td>0.001</td>
</tr>
<tr>
<td>$\gamma = 50$</td>
<td>0.15</td>
<td>-19.84</td>
<td>0.36</td>
<td>25.38</td>
<td>0.53</td>
<td>19.70</td>
<td>21.91</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\gamma = 100$</td>
<td>0.07</td>
<td>-23.22</td>
<td>0.21</td>
<td>30.62</td>
<td>0.26</td>
<td>23.75</td>
<td>27.18</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table 2: Numerical simulations
Columns 1 to 4 report the percentage change of the productivity cut-offs, while column 5 refers to the percentage change in the overall average productivity of country H. Then column 6 and 7 report the wage ratio between the Home and Foreign country $\epsilon$ and the real welfare-based exchange rate RER, and in the end, column 8 shows how the standard deviation of the Pareto distribution function changes when $\gamma$ is changed.

The calibration of parameters follows the benchmark set by Ghironi and Melitz (2005). The choice of $\gamma$ and $\sigma$ is such that the standard deviation of log of US plant sales is equal to 1.67 as Bernard, Eaton, Jensen and Kortum [BEJK 2003] report in their article. Remember that the productivity of firms is Pareto-distributed and the standard deviation of log of sales is exactly equal to $\frac{1}{\gamma - \sigma + 1}$. I will take $\sigma = 3.8$ as reported by [BEJK 2003] on macro trade data and therefore set $\gamma = 3.4$ while the iceberg transport cost is equal to $\tau = 1.3$ such as in Obstfeld and Rogoff (2001). About the fixed cost $F_D$, at general equilibrium the results don’t depend on the level of $F_D$, therefore, without loss of generalization, I can set $F_D = 100$ in order to ensure that cut-offs are greater than the support of the distribution $b$ for every value of $\gamma$. The fixed cost of exporter firms is equal to $(1 + \delta)F_D$ and what is really important here is the share $\delta$ of the fixed cost that Home firms have to pay in order to enter the Foreign market. In the benchmark I set $\delta = 0.1$ as this value gives me a level of openness for the country H which is quite reasonable as the ratio $\frac{EXP}{GDP^H}$ is equal to 11%. This is the actual ratio of exports over the total production although in this model there is only a tradable sector; to calculate the right level of this ratio, I assumed that the share of tradable goods over the total GDP of country H is equal to $1/4$, a value which is often chosen in Macroeconomic literature.

One property of the Pareto distribution function of productivity says that when the shape parameter $\gamma$ is high, firms are less heterogeneous and less productive, that is more distributed at the bottom of the scale of productivity. In my numerical simulations, I want to compare my benchmark analysis with the case of greater homogeneity between firms so to reproduce the framework with one representative firm (when $\gamma \to \infty$) as in Obstfeld and Rogoff’s papers on the adjustment of the current account.

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8I simulate the adjustment of the current account with a level of $\delta = 0.5$ and with an iceberg transport cost $\tau = 1.5$. The results go in the same direction of the benchmark case and the magnitude of the percentage change is bigger than in the benchmark case; for example, with $\delta = 0.5$, the exchange rate depreciates by 1.34% while the average productivity is increased by 0.39%
In order to control for the decrease in productivity when $\gamma$ is high, I set the support of the distribution $b = \frac{1}{\gamma}M$ where $M$ is the average of the distribution function which is exogenously fixed at $M=1$. Then, if you calculate the average of the distribution and its standard deviation you will have:

$$\text{mean} = M \quad \text{and} \quad \text{std.dev.} = \frac{M^2}{\gamma(\gamma - 2)}$$

As a consequence, when I run numerical simulations with a value of $\gamma$ higher than in benchmark, I allow for a level of the standard deviation of the productivity distribution which is lower than before while the average of the distribution does not change and stays equal to $M$. This is due to the automatic increase in the support $b$, depending himself on $\gamma$. For this reason, the standard deviation of the productivity distribution appears in column 8.

The calibration with the benchmark values of parameters implies a level of $T$ such that the CA is adjusted through the direct effect of $T$ on income, so reducing IMP and raising EXP, and through its indirect effect on the exchange rate that must depreciate by 1.22 while the real welfare-based exchange rate is appreciating by 0.12% as the Home consumer-based price index has grown more than the Foreign consumer-based price index.

Once the Home country transfers an amount of resources equal to 7% of its GDP, the productivity of the last domestic firm which enters the Home market has to increase by 1.58% as now these firms has to be more productive to enter the Home market while less productive firms are forced to exit and stay inactive. This is prettily due to the income of Home households which has been lowered by the transfer so reducing the demand for domestic-produced goods. If we look at the exporter firms, the productivity of the last domestic firm which enters the Foreign market declines by 1.80% as the transfer and the depreciation of the Home currency reduces the required cut-off on productivity to enter the Foreign market. In the Foreign country, the results go in the opposite direction than in Home country as the productivity of the last foreign firm which produces for the Foreign market has to decrease by 0.18% while, among the Foreign exporters, there is a massive exit of less productive firms as the threshold goes up by 3.26%.

The overall average productivity of the Home country, which includes all Home producers, both domestic-oriented firms and exporters, increases as a consequence of the adjustment of the current account. Actually, this average is weighted by the share of sales of domestic firms over $GDP^H$ and the share of sales of exporters over $GDP^H$. After the transfer of resources, the share
of export sales increases but still stay behind the share of domestic sales, therefore the decrease in the productivity of exporters accounts less than the increase in the productivity of domestic-oriented firms and so the overall average productivity increases by 0.15%. This result is consistent with the idea that an increase in the average productivity of the country running a deficit in the current account and a decrease in the average productivity of the rest of the world could help the soft-land adjustment of its current account\textsuperscript{9}. The raws following the benchmark case refer to simulations ran with a higher level of $\gamma$. This means that firms are more homogeneous and less productive, so just a few of them can pay the fixed costs and enter the Foreign market: the value of effective traded goods decreases and the Home economy becomes less open. It is straightforward that when firms are more homogeneous and less productive, a higher depreciation by the exchange rate is needed for the adjustment. As a result, in a model where all firms are potentially exporters, I can show that the impact of the extensive margin on the adjustment of the current account is strongly affected by the productivity and the level of heterogeneity of firms. Once I allow for more homogeneity, the extensive margin plays a minor role in the growth of total exports and the real exchange rate has to depreciate even further as in Obstfeld and Rogoff’s (2004) results.

I ran some simulations with a higher level of $\sigma$ and results evidence the presence of the same trend on the depreciation of the exchange rate when firms are more homogeneous, and this depreciation is greater than before although goods are more substitutable. This comes from the higher level of $\gamma$ that I have considered in Benchmark II in order to set the standard deviation of log US sales at 1.67 as reported by [BEJK 2003].

5 Conclusion

The aim of this paper is to find new theoretical evidence on the impact that an adjustment of the current account through a transfer of resources from the

\textsuperscript{9}Remember that in this model with just one tradable sector, an increase in the average productivity of the country corresponds to an increase in the average productivity of the tradable sector. Moreover, in their two-sector model with representative firms producing either one tradable or one non-tradable good, Obstfeld and Rogoff (2004) point out that the rise in the productivity of the Foreign non-tradable producers could make the adjustment of the US CA easier, while, if also Foreign exporters are more productive, their products are cheaper for Home households and the current account position should be eventually worsened.
debtor country toward the rest of the world should have in terms of currency depreciation, productivity thresholds and average sector productivity as well as real welfare-based exchange rate.

The main innovation that this paper brings to the subject of the adjustment of the external account and the transfer debate is the introduction of firms’ heterogeneity.

Firms’ heterogeneity has been of great interest since the paper of Melitz (2003) which has been the landmark for following articles recently published on the New new theory of trade and papers on the decision to export versus the FDI (Helpman, Melitz, Yeaple, 2004). Data actually confirm that only most productive firms export and that the aggregate of exports is raised mainly by the export of new varieties that were not exported before.

I think that a new important challenge for economic scholars is the application of the firms heterogeneity on the field of International Macroeconomics, as only a few papers like Ghironi and Melitz (2005) and Corsetti, Martin, Pesenti (2005) have studied respectively the introduction of heterogeneity between firms and the role of extensive margin on Macroeconomic dynamics. In this perspective, this paper on US current account adjustment with a simple model of firms’ heterogeneity studies the impact that such adjustment has on the level of productivity cut-off of exporters and local-oriented firms both in the Home and in the Foreign country and shows that the extensive margin of trade plays a key role in the adjustment and implies a lower depreciation of the real exchange rate (about 1%) with respect to models with representative firms.

The introduction of the measure of average productivity weighted by the sales of domestic and exporter firms gives a new explanation of the effect that an exogenous shock on the current account like the transfer of resources has on the average productivity of the debtor country and the rest of the world. This can introduce a new element of analysis to the new growing literature of Real Business Cycle models with endogenous entry and exit of firms as the average productivity of firms is not affected by an exogenous given shock but it is endogenously determined by the shock on the current account adjustment.

Further research in my agenda will be addressed to a measure of the average productivity in a framework of firms’ heterogeneity which could better represent the statistically observed measure of the average productivity reported in national accounting. Then it should be important to do some more realistic hypothesis about the portfolio of claims held by householders, which is
well-diversified in this model, and the structure of the economy with just one tradable sector. Nevertheless, the flexibility of this model allows the extension to a completely non-tradable sector and also the introduction of more detailed and realistic hypotheses on the way firms finance their fixed costs. This would allow us to draw new findings on the impact that the adjustment of the current account has on the economy of the United States at a better level of interpretation.
References add Ghironi Melitz Bilbiie


